ISTS-2017-d-063／ISSFD-2017-063

Dynamics and Control of Modular and Extended Space Structures in Cislunar Environment

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(Received April 17th, 2017)

Proposed future space programmes, which, among others, include a space station in lunar vicinity, pose some interesting research problems in the field of non-Keplerian dynamics. This paper investigates the orbit-attitude dynamics and the control of rotational motion of an extended space structure in cislunar environment. The paper presents a fully coupled model for orbit-attitude dynamics, which is based on a Circular Restricted Three-Body Problem formulation. The equations of motion take also into account the most relevant perturbing phenomena, such as the Solar Radiation Pressure (SRP), the fourth-body (Sun) gravity and the variation in the gravitational attraction due to the finite dimension of the large space structure. Preliminary results exploiting efficient control methods are presented. Single and dual-spin stabilisation are compared and the results are carefully analysed to highlight a control strategy that is less resource consuming. The space of orbit-attitude solutions is studied to highlight possible stable conditions that may be exploited to host the cislunar station with minimum control effort. The outcomes of the research presented in this paper are intended to highlight drivers for the lunar outpost design and station-keeping cost minimisation. Furthermore, a case study for a large space structure in selected non-Keplerian orbits around Earth-Moon collinear Lagrangian points is discussed to point out some relevant conclusions for the potential implementation of such a mission.

Key Words: Orbit-attitude dynamics, Non-Keplerian orbits, Cislunar large space structures, GNC

1. Introduction

Future space exploration missions will be less and less confined to Low-Earth orbits. In fact, the attention of the international space community is evolving towards cislunar and interplanetary space[7] . The modern vision about this broad exploration program is based on the sustainability of the entire network of systems and operations to accomplish the established ambitious goals, which would not be achievable without a solid support of preliminary missions, intermediate steps and new technologies development. Planned human and robotic exploration of the Solar System will rely on a complex infrastructure of automated transfer vehicles, space stations and logistics operations that will be progressively ideated and developed. The feasibility of the whole project is strongly dependent from the improvements in new trajectory design and GNC techniques that have to leverage Three-Body problem dynamics, coupled orbit-attitude equations of motion, appropriate structural models and efficient control techniques. These enhanced methods are especially needed when dealing with a large and flexible space structure, such as a space station in the vicinity of the Moon: key point for the successful realisation of the aforementioned exploration program.

The analysis of this system is in a preliminary phase, and the architecture still have to be defined, but it is already clear that the cislunar station will be assembled in-orbit by means of many automated operations. They will be carried out in a non-Keplerian environment, being one of the Earth-Moon libration points the ideal location for a space system of this kind.[12] Indeed, for instance, orbits about one of the Earth-Moon collinear libration points, such as EML (Earth-Moon Lagrangian Point) Halo orbits, have continuous line of sight coverage for communications and they can be easily accessed from the Earth with existing transportation systems. In addition, other families of non-Keplerian orbits have appealing characteristics, such as the excellent orbit stability of Distant Retrograde Orbits (DRO) or the satisfactory ease of access from the Moon of Near-Rectilinear Orbits (NRO). In this paper, all the aforementioned families of orbits are considered and analysed.

The most appropriate method to analyse an extended space system in cislunar environment is founded on the well-known Circular Restricted Three-Body Problem (CR3BP) and expressed through a coupled orbit-attitude model. In fact, while the majority of existing literature in this research context is founded on dynamical models based on point-mass dynamics without the inclusion of the rotational motion, accurate analysis on a large space system can be carried out only taking into account also the attitude dynamics. Moreover, the model used in this research work includes the most relevant perturbing phenomena, such as the second order deviations in the main gravitational attraction due to the finite extension of the spacecraft, the Solar Radiation Pressure and the fourth-body (Sun) gravity.

When the coupled orbit-attitude dynamics is investigated in a non-Keplerian environment, the translational and rotational behaviours of space systems may have extremely complex evolutions. However, the chaotic appearance, which is typical when more than one massive primary body is considered, hides some interesting dynamical structures that may be exploited to design space missions, leveraging the orbit-attitude dynamics to satisfy very complicated requirements. For example, naturally periodic orbit-attitude solutions could enable coarse pointing operational modes for data acquisition or communications without a relevant control action. Yet in addition, a simple attitude control strategy could dramatically increase the design freedom, the pointing capabilities, the rendezvous and docking possibilities and the easiness of rotation manoeuvring. Indeed, the possibility to have an additional degree of freedom on the attitude dynamics allows enhanced operations. In this paper, spin stabilisation methods are exploited to simply control the space systems with a limited active consumption of resources.

First investigations about attitude dynamics in the restricted three-body problem assumed the spacecraft as artificially maintained close to the equilibrium points and only the stability of the motion was considered[8, 11) . Afterwards, Euler parameters were introduced to study the rotational dynamics of a single body located at one of the Lagrangian point.[1) More recently, other authors focused their attention to the attitude dynamics of a spacecraft in the vicinity of equilibrium points, using Poincaré maps and linear approximations of non-Keplerian orbits[2, 13) .

In the last few years, the coupling between orbital and attitude motion was investigated considering both planar and full three-dimensional motion, providing different families of orbit-attitude solutions[6, 9) . In the same years, a stability analysis of dual-spin spacecrafts in non-Keplerian orbits with semi-analytical approach was proposed, underlining certain modes of motion.[10)

Most recently, Colagrossi and the research group at Politecnico di Milano developed a model to study fully coupled orbit-attitude perturbed motion in three-dimensional and planar space, with applications to various scientific and technological objectives[3, 4) . In particular, the natural orbit-attitude dynamics of an extended and flexible space structure has been investigated drawing general preliminary conclusions about this typology of dynamical system[5) .

The paper starts introducing the coupled model for orbit-attitude dynamics, which is based on a Circular Restricted Three-Body Problem formulation with the addition of the previously mentioned perturbing effects. Subsequently, the attitude spin stabilisation methods are introduced and applied on some reference periodic motions. Then, the stability properties of different orbit-attitude dynamics are critically analysed and relevant implications for operations are highlighted. Lastly, representative solutions are illustrated and discussed, with particular attention to the case study of an extended space structure in non-Keplerian orbits around Earth-Moon collinear Lagrangian points.

2. Orbit-attitude dynamical model

The orbit-attitude dynamical model is founded on Circular Restricted Three-Body Problem modelling approach, which consider the motion of three masses m_1, m_2 and m, where $m \ll m_1, m_2$ and $m_2 < m_1$. m_1 and m_2 are denoted as primaries, and are assumed to be in circular orbits about their common centre of mass. The body m does not affect the motion of the primaries.

The translational dynamics of m is conveniently expressed in the rotating synodic reference frame, S, which is shown in fig. 1. It is centred at the centre of mass of the system, O; the $\hat{\mathbf{x}}$ axis, is aligned with the vector from m_1 to m_2; the third axis, $\hat{\mathbf{z}}$, is in the direction of the angular velocity of S, $\boldsymbol{\omega} = \omega\,\hat{\mathbf{z}}$; $\hat{\mathbf{y}}$ completes the right-handed triad. At $t = 0$, the rotating frame S is aligned to the inertial frame I, which is centred in O and is defined by the versors $\hat{\mathbf{X}}$, $\hat{\mathbf{Y}}$ and $\hat{\mathbf{Z}}$.

The system is defined by the mass parameter,

$$\mu = \frac{m_2}{m_1 + m_2},$$

the magnitude of the angular velocity of S,

$$\omega = \sqrt{\frac{G(m_1 + m_2)}{r_{12}^3}},$$

and the distance between the primaries r_{12}. The equations of motion are usually normalised such that r_{12}, ω and the total mass of the system, $m_T = m_1 + m_2$, are unitary in non-dimensional units. These units are indicated with the symbol [nd] in the paper. As a consequence, after the normalisation, the period of m_1 and m_2 in their orbits about their centre of mass is $T = 2\pi$. In this paper, the parameters that have been used to normalise the equations of motion in the Earth-Moon system are $r_{12} = 384\,400$ km, $m_T = 6.04 \times 10^{24}$ kg and $T = 2\pi/\omega = 27.28$ d.

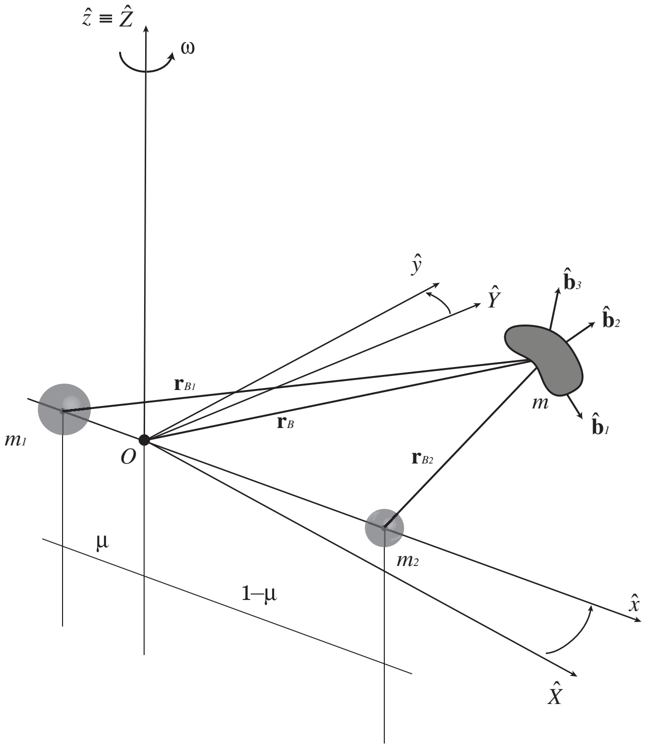


Figure1.: Synodic and inertial reference frames.

The orbital dynamics of the body m has been modelled considering the usual Circular Restricted Three-Body Problem formulation, valid for point-mass unperturbed dynamics, plus the contribution of the Solar Radiation Pressure, the fourth-body gravity and the variation in the gravitational attraction due to the finite dimension of m, expressed with the second order term of the force exerted on a finite dimension body by a particle.

The resulting problem is written in the following normalised scalar form:

$$\mathbf{f}_x = \begin{cases} \dot{x} = v_x \\ \dot{y} = v_x \\ \dot{z} = v_z \end{cases} \tag{1}$$

$$
\mathbf{f}_v = \begin{cases} \dot{v}_x = x + 2v_y - \dfrac{(1-\mu)(x+\mu)}{r_{B_1}^3} - \dfrac{\mu(x-1+\mu)}{r_{B_2}^3} \\ \qquad + a_{1_x} + a_{2_x} + a_{SRP_x} + a_{4th_x} \\ \dot{v}_y = y - 2v_x - \dfrac{(1-\mu)y}{r_{B_1}^3} - \dfrac{\mu y}{r_{B_2}^3} \\ \qquad + a_{1_y} + a_{2_y} + a_{SRP_y} + a_{4th_y} \\ \dot{v}_z = -\dfrac{(1-\mu)z}{r_{B_1}^3} - \dfrac{\mu z}{r_{B_2}^3} \\ \qquad + a_{1_z} + a_{2_z} + a_{SRP_z} + a_{4th_z}, \end{cases} \tag{2}
$$

where x, y and z are the Cartesian coordinates of O_B expressed in terms of the synodic reference frame; v_x, v_y and v_z are the velocity components of the body m in S. The distances between the centre of mass of m and the two primaries are respectively $r_{B_1} = \sqrt{(x+\mu)^2 + y^2 + z^2}$ and $r_{B_2} = \sqrt{(x-1+\mu)^2 + y^2 + z^2}$, as can be easily noted from fig. 1. The terms $a_{1_{x,y,z}}$, $a_{2_{x,y,z}}$, $a_{SRP_{x,y,z}}$ and $a_{4th_{x,y,z}}$ are, respectively, the normalised accelerations due to the variation in the gravitational attraction of the first and second primary due to the finite dimension of m, to the Solar Radiation Pressure and to fourth-body (Sun) gravity

The body m is extended and three-dimensional. Hence, it has six degrees of freedom: the position of its centre of mass in S, which is easily described by the position vector $\mathbf{r}_B$, and the orientation of the body reference frame B with respect to I or S. The four-dimensional quaternion unit vector, $\mathbf{q}_B^I = [q_1, q_2, q_3, q_4]^\mathrm{T}$, also known as Euler parameters, is used as attitude parameter that relates the frame B with respect to the frame I. Indeed, in the present model, the equations of rotational motion are written in the inertial frame. The body-fixed frame B is centred at the centre of mass of m, O_B, and it is aligned with the body principal inertia directions, $\hat{\mathbf{b}}_1$, $\hat{\mathbf{b}}_2$ and $\hat{\mathbf{b}}_3$.

The resulting attitude kinematics and dynamics equations are:

$$
\mathbf{f}_q = \begin{cases} \dot{q}_1 = \frac{1}{2}(\omega_1 q_4 - \omega_2 q_3 + \omega_3 q_2) \\ \dot{q}_2 = \frac{1}{2}(\omega_1 q_3 + \omega_2 q_4 - \omega_3 q_1) \\ \dot{q}_3 = \frac{1}{2}(-\omega_1 q_2 + \omega_2 q_1 + \omega_3 q_4) \\ \dot{q}_4 = -\frac{1}{2}(\omega_1 q_1 + \omega_2 q_2 + \omega_3 q_3) \end{cases} \tag{3}
$$

$$
\mathbf{f}_\omega = \begin{cases} \dot{\omega}_1 = \dfrac{I_3 - I_2}{I_1}\Big(\dfrac{3(1-\mu)}{r_{B_1}^5} l_2 l_3 + \dfrac{3\mu}{r_{B_2}^5} h_2 h_3 \\ \qquad - \omega_2\omega_3\Big) + \alpha_{SRP_1} + \alpha_{4th_1} \\ \dot{\omega}_2 = \dfrac{I_1 - I_3}{I_2}\Big(\dfrac{3(1-\mu)}{r_{B_1}^5} l_1 l_3 + \dfrac{3\mu}{r_{B_2}^5} h_1 h_3 \\ \qquad - \omega_1\omega_3\Big) + \alpha_{SRP_2} + \alpha_{4th_2} \\ \dot{\omega}_3 = \dfrac{I_2 - I_1}{I_3}\Big(\dfrac{3(1-\mu)}{r_{B_1}^5} l_1 l_2 + \dfrac{3\mu}{r_{B_2}^5} h_1 h_2 \\ \qquad - \omega_1\omega_2\Big) + \alpha_{SRP_3} + \alpha_{4th_3}, \end{cases} \tag{4}
$$

where I_1, I_2 and I_3 are the principal moments of inertia of m and ω_1, ω_2 and ω_3 are components of the angular velocity of the body relative to I, expressed in the body-fixed reference frame B, $\boldsymbol{\omega}_B^I$. The gravity gradient torques due to the primaries are evaluated in terms of direction cosines l_i and h_i, which are respectively computed in the reference B for the unit position vectors $\hat{\mathbf{r}}_{B_1}$ and $\hat{\mathbf{r}}_{B_2}$. The components of the angular accelerations due to the SRP and to the presence of the Sun are expressed as $\alpha_{SRP_{1,2,3}}$ and $\alpha_{4th_{1,2,3}}$.

The effects due to the structural flexibility of the large spacecraft have not been considered in this paper since, in the previous works of the authors, the coupling between orbit-attitude dynamics and flexible dynamics has resulted to be not so relevant for typical frequencies of the non-Keplerian dynamics. Moreover, the assumption is valid also with the control strategy proposed in this work. In fact, as will be discussed in the following, the attitude stabilisation does not change the frequency separation between orbit-attitude dynamics and the lowest natural frequencies of real extended space systems.

The complete set of non-linear differential equations, describing the coupled orbit-attitude dynamics of an extended body in cislunar environment, is therefore the collection of eqs. (1) to (4) denoted as $\mathbf{f} = \{\mathbf{f}_x, \mathbf{f}_v, \mathbf{f}_q, \mathbf{f}_\omega\}$.

Further details on the dynamical model described in this section, such as the complete definition of the perturbing terms, or the numerical techniques that are exploited to find periodic solution, can be found in the paper that is dedicated to describe some preliminary results on the dynamics of extended bodies in cislunar space[5)] .

3. Attitude control strategy

Naturally periodic orbit-attitude motions in cislunar environment do exist and, in some case, they are also remarkably stable. However, an attitude control strategy can be desired to be actuated on-board of an extended space structure in lunar vicinity for a series of reasons. For instance, there might be the need to increase the stability of a certain configurations, or to manoeuvre the spacecraft or even to bring the system in unstable conditions with the purpose to excite some natural dynamics able to drive large slewing manoeuvres. The constraint that should be enforced while designing the attitude control system imposes a strong limitation in active consumption of resources while controlling the dynamics. In fact, the cislunar space station will be operative for a long time, with limited supplies and conservative power budget, relatively to its dimensions and its tasks to be accomplished. For these reasons, this research work focus its attention on single-spin or dual-spin attitude control techniques, which are more precisely referred to as attitude stabilisation techniques. Indeed, the stabilisation action with spinning spacecraft or with constant speed momentum wheels is remarkably efficient in terms of energy consumption.

Spin stabilisation techniques are based on the gyroscopic effect of the angular momentum stored within the body m. In single-spin stabilisation method, the whole spacecraft is spinning and the rotating mass of the spacecraft acts as attitude stabilising system. While, dual-spin stabilisation methods are based on momentum wheels that are able to store an important amount of angular momentum, needed to stabilise the system.

Nevertheless, they can have a different rotation speed with respect to the main body and, thus, there is one additional degree of freedom that can be exploited while designing the mission operations. Furthermore, momentum wheels can be easily controlled in spinning rate or direction, and this feature opens to the possibility of attitude manoeuvres and enhanced control capabilities.

The additional stored angular momentum, which is eventually due to the presence of momentum wheels, affects the dynamic of the system as if the internal angular momentum were:

$$\mathbf{h} = \mathbf{I}\omega_B^I + \mathbf{h}_w, \tag{5}$$

where $\mathbf{I}$ is the inertia tensor of the spacecraft, which takes into account the moments of inertia of the momentum wheels, and $\mathbf{h}_w$ is the angular momentum of the momentum wheels expressed in body reference frame. Assuming the presence of three different momentum storage devices, aligned with the principal axes of the body m, the angular momentum of the wheels is:

$$\mathbf{h}_w = [I_{1_w}\omega_{1_w}, I_{2_w}\omega_{2_w}, I_{3_w}\omega_{3_w}]^{\mathrm{T}}, \tag{6}$$

where I_{1_w}, I_{2_w}, I_{3_w} are the moments of inertia of the rotors respectively aligned with $\hat{\mathbf{b}}_1$, $\hat{\mathbf{b}}_2$ and $\hat{\mathbf{b}}_3$; ω_{1_w}, ω_{2_w} and ω_{3_w} are the relative angular velocities of three momentum wheels with respect to the body frame.

Therefore, eq. (4) has to be modified with the additional terms due to the presence of the rotating momentum wheels that can be evaluated as described in classic literature about rigid body dynamics:

$$\boldsymbol{\eta} = \omega_B^I \times \mathbf{h}_w = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \begin{bmatrix} I_{1_w}\omega_{1_w} \\ I_{2_w}\omega_{2_w} \\ I_{3_w}\omega_{3_w}. \end{bmatrix} \tag{7}$$

The three components, η_1, η_2 and η_3 of the vector $\boldsymbol{\eta}$ are then respectively divided by the moments of inertia of the body m. The resulting terms η_1/I_1, η_2/I_2 and η_3/I_3 are successively subtracted from the right-hand side of eq. (4). The momentum wheels are assumed to be operated with constant spinning rate and axis and, thus, no additional term, such as derivative of the angular momentum of the rotors $\dot{\mathbf{h}}_w$ has to be included in the present model. Moreover, the additional equations of motion for the momentum wheels in $\mathbf{f}$ are trivial, being:

$$\dot{\omega}_{i_w} = 0 \longleftrightarrow \omega_{i_w} = const, \quad \text{with} \quad i = 1, 2, 3. \tag{8}$$

It must be noted that the differential correction scheme described in the previous works of the authors[5)] should be slightly modified. In fact, the Jacobian of the system now contains the terms due to the presence of the momentum wheels. Hence, the State Transition Matrix is a bit different from the usual coupled orbit-attitude dynamical model. On the contrary, the constraint vector is unmodified because the periodicity is not sought in the dynamics of the momentum wheels. All the relevant details are thoroughly described in the cited reference.

Single-spin attitude stabilisation can be analysed with the model described in this paper, just considering the momentum wheels as non-rotating devices or as zero inertia rotors (e.g. $\omega_{i_w} = 0$ or $I_{i_w} = 0$ with $i = 1, 2, 3$).

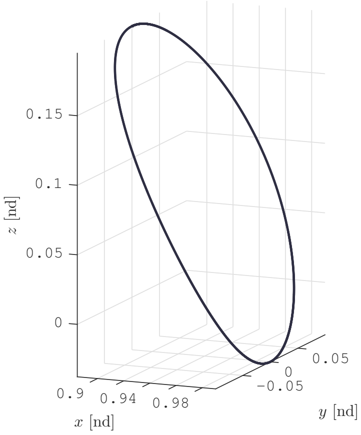
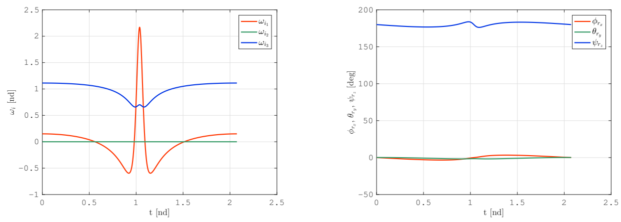


Figure2.: Reference EML1 NRO Orbit: $T_{NRO} = 2.07\,\text{nd} = 8.98\,\text{d}$, $A_{z_{Max}} = 0.195\,\text{nd} = 7.49 \times 10^4\,\text{km}$.



(a) Angular rates. (b) Euler angles.

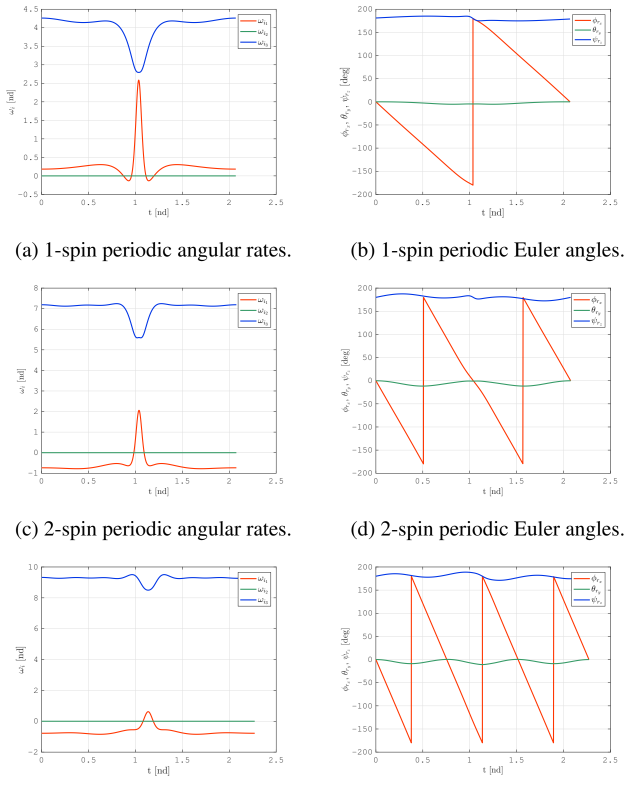
Figure3.: Periodic librating (0-spin) attitude dynamics on the reference EML1 NRO Orbit (fig. 2): $k_3 = 0.2$.

4. Single-spin attitude stabilisation

The single-spin attitude stabilisation is a very simple and effective technique to increase the stability of the rotational motion. Even though the details about attitude stability will be discussed later in the paper, the general characteristics of single-spin periodic dynamics are introduced in this section.

Periodic orbit-attitude motion in cislunar environment has to satisfy periodicity constraints in both orbital and attitude variables and, moreover, the attitude evolution should be compatible and periodic with the gravity gradient torques due to the presence of the primaries. In fact, the effects of the gravitational attraction on the rotational motion strongly characterises the periodic dynamics. This is true in particular for non-Keplerian orbits with a low perilune altitude with respect to the lunar surface, such as large amplitude Lyapunov orbits, elongated Halo orbits or NROs. The latter are among the ones with the lowest perilune passage and, therefore, experience a large gravity gradient torque that determines a relevant angular acceleration on the extended body, which is a source of instability for the attitude dynamics. For this reason, a NRO around the Earth-Moon L1 point (EML1) is used as a reference orbit to analyse the features of single-spin attitude stabilisation. Details about the reference NRO are reported in fig. 2.

Several attitude periodic motions are possible on the same orbit, but only few examples are shown here to highlight the general features of the stabilised orbit-attitude periodic motions. The reference dynamics is the one that is not spin stabilised, being just librating around the equilibrium position. As can



(a) 1-spin periodic angular rates. (b) 1-spin periodic Euler angles.

(c) 2-spin periodic angular rates. (d) 2-spin periodic Euler angles.

(e) 3-spin periodic angular rates. (f) 3-spin periodic Euler angles.

Figure4.: Periodic single-spin attitude dynamics on the reference EML1 NRO Orbit (fig. 2): $k_3 = 0.2$.

be seen in fig. 3, the body m performs zero overall rotations in synodic frame (fig. 3b) and the librational motion is due to the gravity gradient torque of the primaries (other perturbing phenomena are not considered in this section). The presented solution has been obtained for a body with inertia parameter $k_3 = \frac{I_2 - I_1}{I_3} = 0.2$. In fig. 3a, the noticeable angular acceleration at the perilune passage is evident in the middle of the simulation. In that point, the spacecraft must be correctly oriented in order to remain stable on the naturally periodic motion, otherwise the large gravity gradient torque, different from the nominal one, generates the unstable dynamics. Moreover, since the spacecraft has not stored angular momentum, a fast departure from the reference condition is likely to happen. The angular velocity reported in the figures is evaluated with respect to the inertial reference frame, and the component along $\hat{\mathbf{z}}$ has an offset of 1 in non-dimensional units because of the rotational motion of the synodic frame with respect to the inertial one. Thus, a velocity component $\omega_{I_z} = 1$ nd in inertial frame is equivalent to $\omega_{S_z} = 0$ nd in synodic frame.

The single-spin dynamics on the reference EML1 NRO are shown in fig. 4. These all have similar features because of the attitude that is initialised as in the reference librating solution and the inertia parameter of the body is $k_3 = 0.2$. The only difference is the spinning rate around $\hat{\mathbf{b}}_3$ that, in the first case in fig. 4a, allows one overall rotation along one orbit, while in the other two simulations in figs. 4c and 4e determines, respectively, two and three overall rotations in one orbital period. The angular accelerations due to the gravity gradient of the Moon is evident also in these situations, but the effect is weaker if compared to the global magnitude of the angular velocity. Furthermore, the increasing stored angular momentum makes the spinning body less influenced by external perturbations and the resulting attitude dynamics more stable and stiff. The increase in attitude stability will be formally discussed in the following but, from a general overview of the attitude evolution for increasing spinning rates, presented in fig. 4, it is evident how the attitude dynamics is less affected by the gravity gradient torque at perilune. On the other hand, such a spinning condition, may be problematic in terms of operations and other mission constraints. For example, telecommunications or docking activities may be more complex in the case the cislunar station is rotating with large angular rate.

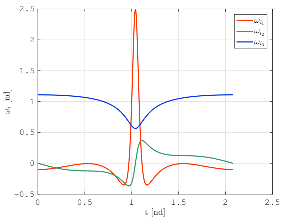
Single-spinning attitude stabilisation solutions are available also for other orbital families. For instance, single-spin attitude solutions on DROs have been discussed in a previous works of the authors[5)] , but they are not reported here because the large stability of DROs also in the librating solution, together with the very small gravity gradient torque generated by the primaries, makes them not so interesting in terms of attitude control. Nevertheless, if enhanced stability properties or different operational requirements are sought along DROs, single-spin stabilisation is possible and periodic spinning solutions are easily available.

5. Dual-spin attitude stabilisation

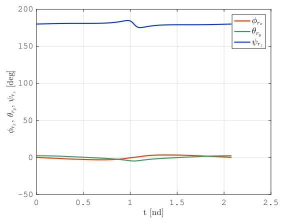
The operational constraints imposed by the single-spinning attitude stabilisation methods can be easily overcome with a separate angular momentum storage device, which can be spun at a different angular rates with respect to the main body. Thus, there is one additional degree of freedom that can be exploited to stabilise the attitude dynamics without inserting additional operational constraints. In this research work, the spinning momentum wheels mounted on the body m are used at a constant spinning rate. The increased angular momentum of the whole space system is the foundation for the dual-spin attitude stabilisation technique.

The periodic solutions shown in fig. 5 refer to three distinct angular rates of the spinning momentum wheel, leading to as many dual-spin attitude periodic dynamics. They have been initialised on the reference EML1 near rectilinear orbit, starting from the librating attitude solution in fig. 3 and considering a body with inertia parameter $k_3 = 0.2$. Hence, they all share comparable characteristics in order to correlate the two attitude stabilisation methods. In the analysed dual-spin solutions there is only one spinning momentum wheel, which is the one along the principal inertia axis $\hat{\mathbf{b}}_3$. In this way, there is a direct connection between the single-spin and the dual-spin with spinning direction along the same body axis. Furthermore, an increased angular momentum along $\hat{\mathbf{b}}_3$ is the one that is needed to make possible and stabilise the attitude dynamics for the given orbit and initial orientation. The momentum wheel has moment of inertia $I_{3_w} = \frac{I_3}{100}$.

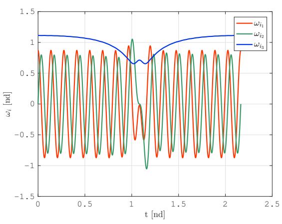
The three proposed periodic solutions differ for the spinning rate of the momentum wheel. In the first case, fig. 5a, the wheel is slowly spinning with $\omega_{3_w} = 500$ nd and the stabilisation effect is not so evident, except for the additional rotational motion around $\hat{\mathbf{b}}_2$ due to the gyroscopic coupling. The dual-spin behaviour starts to be more evident with an higher angular veloc-



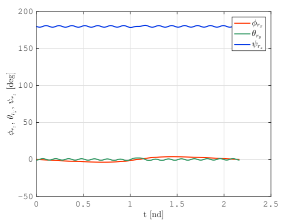
(a) Slow spinning wheel periodic angular rates (ω_{3_w} = 500 nd).



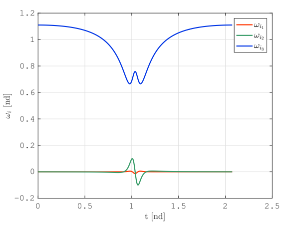
(b) Slow spinning wheel periodic Euler angles (ω_{3_w} = 500 nd).



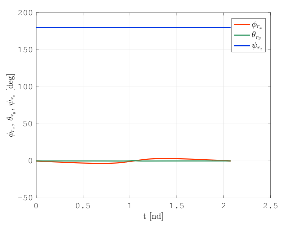
(c) Medium spinning wheel periodic angular rates (ω_{3_w} = 5000 nd).



(d) Medium spinning wheel periodic Euler angles (ω_{3_w} = 5000 nd).



(e) Fast spinning wheel periodic angular rates (ω_{3_w} = 50 000 nd).



(f) Fast spinning wheel periodic Euler angles (ω_{3_w} = 50 000 nd).

Figure5.: Periodic dual-spin attitude dynamics on the reference EML1 NRO Orbit (fig. 2): $k_3 = 0.2$.

ity of the momentum wheel in fig. 5c. In this case, the increased angular momentum makes the system more stiff and the gyroscopic coupling frequency is high, with fast oscillations around the rotation axis of the body. The effect due to the large gravity gradient of the Moon is mitigated and the attitude is stable in its pulsating evolution. However, a great improvement in attitude stabilisation is obtained further increasing the spinning rate of the momentum wheel. In fig. 5e the momentum wheel is spinning at a fast rate, ω_{3_w} = 50 000 nd, and the attitude dynamics is greatly stabilised with limited angular acceleration at the perilune. With dual-spin attitude stabilisation is therefore possible to stabilise the attitude, limiting the effect of the gravity gradient torque, with the body that is no more rotating in the Synodic reference frame. In fact, comparing fig. 5f and fig. 4f, a spinning momentum wheel allows to greatly increase the performance of the attitude dynamics, while maintaining the cislunar station just librating around an equilibrium condition. Thus, no additional mission constraints are imposed and the space system can be operated more easily with respect to the case in which the whole spacecraft is rotating with large angular rate. Moreover, the proposed stabilisation can be practically implemented since the ω_{3_w} = 50 000 nd corresponds in dimensional units to ω_{3_w} = 0.133 rad/s = 1.27 RPM.

6. Attitude stability

At this point, the necessity to introduce a precise definition of attitude stability is needed. In fact, assuming the body on its operational orbit with imposed attitude dynamics in order to achieve a coupled periodic motion, it is interesting to quantify the dynamical properties of the orbit-attitude motion. Many consideration are possible on the coupled stability, as well as the mutations of the dynamical behaviour along a family of orbit-attitude periodic solution (e.g. bifurcations in the dynamical structure of the family of periodic solutions). Additional investigations on this broad section about coupled orbit-attitude dynamics in non-Keplerian environments will follow the results presented in this paper and will be presented in a separate research work. By now, the stability analysis is restricted to the periodic attitude dynamics on a certain reference non-Keplerian orbit.

The considerations presented in this section are based on some outcomes of the Floquet theory, similarly to what has been already done by other authors in previous orbit-attitude and restricted three-body problem literature[6, 10] . In particular, the first consideration is based on the fact that solutions in the vicinity of a periodic reference are linearly approximated by the modes of the monodromy matrix (i.e. the state transition matrix over one period). These linear modes allow to investigate the linear stability properties along the periodic solution.

In order to focus the analysis on the attitude stability, only the attitude part of the monodromy matrix, $\mathbf{M}$, is taken into account (i.e. the submatrix where only attitude variables are involved):

$$\mathbf{M}_{Attitude} = \begin{bmatrix} M_{qq} & M_{q\omega} \\ M_{\omega q} & M_{\omega\omega} \end{bmatrix}. \tag{9}$$

It should be noted that, as explained in the reference literature, the monodromy matrix is transformed into the synodic rotating frame, even though the state variables are expressed with respect to the inertial frame I. Linear attitude modes are therefore associated to the eigenstructure of $\mathbf{M}_{Attitude}$, which is composed by 6 eigenvalues $\lambda_{Attitude_i}$. Those with magnitude less than one are related to linear stable modes, while those with magnitude greater than one correspond to linear unstable modes. Attitude eigenvalues with $\left\|\lambda_{Attitude_i}\right\| = 1$ are paired to marginally stable modes. As a consequence, if $\left\|\lambda_{Attitude_i}\right\| \leq 1$ for any i, the periodic attitude solution is stable (or marginally stable) in the linear approximation. On the contrary, if at least one $\left\|\lambda_{Attitude_i}\right\| > 1$, the periodic solution is unstable.

Furthermore, according to what has been already introduced by previous authors, a stability index, σ, can be defined in order to simplify the stability analyses. In particular, this quantity is defined as:

$$\sigma = \frac{1}{2}\left(\lambda_{Attitude_{Max}} + \frac{1}{\lambda_{Attitude_{Max}}}\right), \tag{10}$$

where $\lambda_{Attitude_{Max}} = \max \left\|\lambda_{Attitude_i}\right\|$ is the magnitude of the dominant eigenvalue. According to this definition, $\sigma = 1$ is associated to marginally stable attitude dynamics, while $\sigma > 1$ represent unstable attitude solution and a larger stability index can be related to a faster departure from the periodic motion. Stable dynamics are associated to $\lambda_{Attitude_{Max}} < 1$.

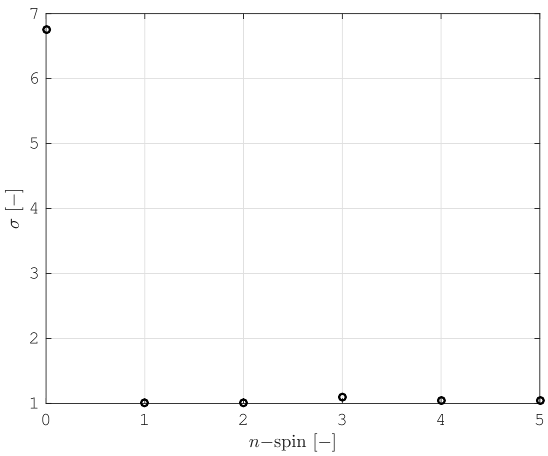


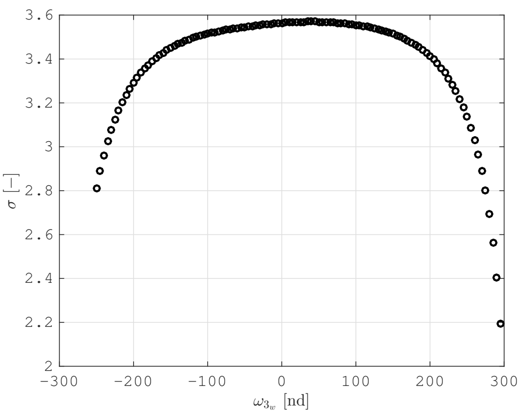
Figure6.: Stability index for single-spin attitude stabilisation solutions (see fig. 4).

Analysing for example the stability index for different single-spin attitude solutions, reported in fig. 6, it is evident the stability improvement due to the spinning stabilisation methods. Indeed, the stability index, which is equal to 6.81 for the librating solution, approaches the value 1 for all the spinning dynamics. Small deviations towards instability are possible, looking for example at the 3−spin periodic motion, but σ is always very close to 1, meaning that the spinning solution departs from the periodic motion slowly.

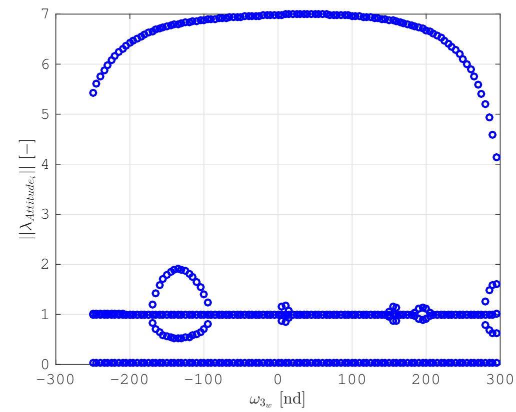
The same analysis is possible for dual-spin attitude stabilisation. In fact, similarly to the figure for single-spin dynamics, the attitude stability increases for an increasing stored angular momentum due to the presence of a faster wheel. However, the progressive evolution of σ along a family of periodic solutions with different angular rates of the momentum wheel is more interesting for dual-spin dynamics.

For instance, fig. 7 reports the stability index and the magnitude of attitude eigenvalues for different dual-spin solutions, $\omega_{3_w} \in [-250, 300\,\mathrm{nd}]$, on a reference EML1 Halo orbit, whose details are reported in fig. 7. From fig. 7a is evident the general increase in stability (i.e. σ decreases) for increasing angular rate of the wheel. Moreover, the evolution of the magnitude of $\lambda_{Attitude_i}$, in fig. 7b, highlights the presence of distinct bifurcation points. These points are associated with a change in the eigenstructure of the periodic solution, as evident in fig. 7b for $\omega_{3_w} \simeq -175\,\mathrm{nd}$, where a saddle point appears in place of two eigenvalues on the unit circle that disappear.

This fact opens to the possibility to facilitate the manoeuvres between different periodic attitude families by varying the amount of stored angular momentum in the wheel. In practice, when the system is at a bifurcation point, a small perturbation in the direction of the desired bifurcating family could enable a variation in the attitude motion. However, it should be noted that the stability properties of the bifurcating solution may be not satisfactory and, thus, the system may be naturally inclined towards the most stable dynamics. In this perspective, the bifurcation points should be just exploited to begin a desired manoeuvre, which has to be correctly driven in order to acquire a precise periodic attitude motion. Furthermore, having in mind that the cislunar space station will be assembled in-orbit, through many docking and undocking operations with massive modules, the inertia parameters of the system can vary in time. Hence, bifurcating solutions may be exploited in order to connect two stable periodic attitude dynamics associated with the different inertia properties of the modular space station.



(a) Stability index, σ.

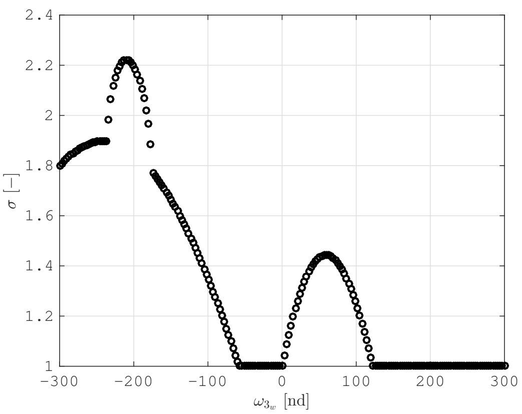


(b) Magnitude of eigenvalues, $\left\|\lambda_{Attitude_i}\right\|$.

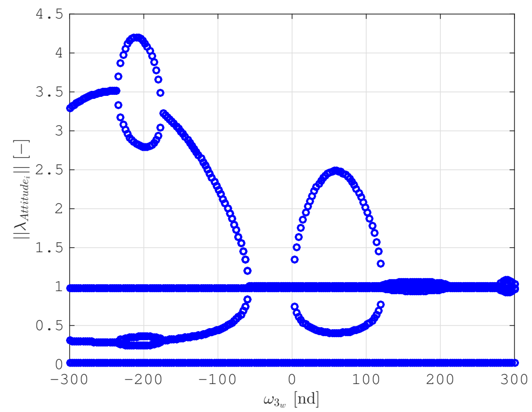
Figure7.: Dual-spin stability analysis for attitude family on reference EML1 Halo orbit: $T_{Halo} = 2.30\,\mathrm{nd} = 9.98\,\mathrm{d}$, $A_{z_{Max}} = 0.18\,\mathrm{nd} = 6.9 \times 10^4\,\mathrm{km}$, $k_3 = 0.2$ and $\omega_{3_w} \in [-250, 300\,\mathrm{nd}]$.

An additional analysis on the considered framework is possible looking at the stability properties of another dynamical family. Figure 8 shows stability index and the magnitude of attitude eigenvalues for several dual-spin solutions, $\omega_{3_w} \in [-300, 300\,\mathrm{nd}]$, on a reference DRO defined in the caption. Distant retrograde orbits are highly stable planar trajectories around both the collinear points L1 and L2, associated to smooth and stable periodic attitude solutions. In general, the whole orbit-attitude dynamics on DROs is remarkably stable, as ca be also understood looking at the stability index in fig. 8a, especially in comparison with the stability analysis for the Halo presented in fig. 7.

In this case, the unstable solution may be of interest with the purpose to excite some natural dynamics able to drive large attitude manoeuvres or fast slewing operations. In fact, when an unstable mode exists it can be excited in order to move the system along a natural trajectory that evolves towards a desired final condition. Dual-spin stabilisation is effective also in this situation, where the space system is orbiting along a DRO with librating rotational motion in synodic reference frame. The angular velocity of the body m associated with this particular

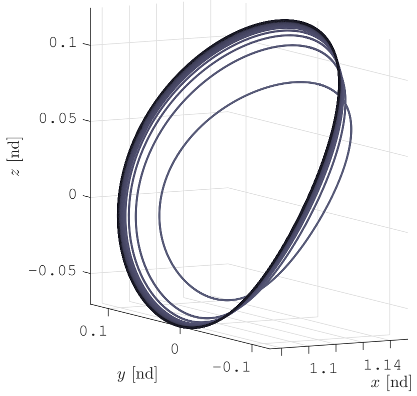


(a) Stability index, σ.

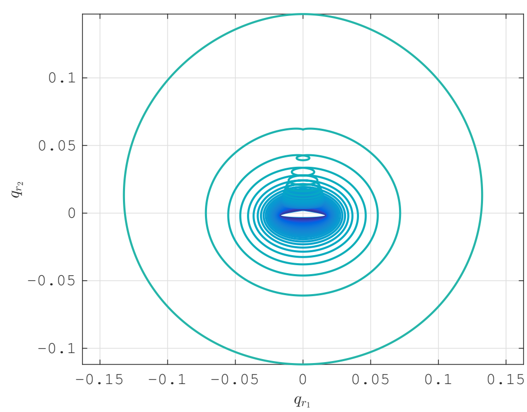


(b) Magnitude of eigenvalues, $\left\|\lambda_{Attitude_i}\right\|$.

Figure8.: Dual-spin stability analysis for attitude family on reference DRO: $T_{DRO} = 3.37\,\text{nd} = 14.63\,\text{d}$, $k_3 = 0.2$ and $\omega_{3_w} \in [-300, 300\,\text{nd}]$.



(a) Orbital trajectories along the family.



(b) Quaternions subspace, components q_1 and q_2 along the family.

Figure9.: Periodic orbit-attitude family on EML2 Halo orbits with dual-spin attitude stabilisation: $T_{Halo_1} = 3.35\,\text{nd} = 14.57\,\text{d}$, $A_{z_{Max_1}} = 0.083\,\text{nd} = 3.18 \times 10^4\,\text{km}$, $k_3 = 0.2$ and $\omega_{3_w} \in [-100, 100\,\text{nd}]$.

attitude dynamics is, in inertial frame, $\omega_3 = 1\,\text{nd}$. Hence, a counter-spinning wheel able to cancel the stored angular momentum of the system can decrease the stability level of the whole system. In fig. 8a, the largest stability index (i.e. the highest instability) is at $\omega_{3_w} = -237\,\text{nd}$, because the inertia of the rotors is $I_{3_w} = \frac{I_3}{237}$. For this particular spinning rate of the momentum wheel, the system has internal angular momentum along $\hat{\mathbf{b}}_3$ equal to zero and, therefore, null gyroscopic stiffness. The resulting motion is not stable and the presence of unstable modes can be exploited for the aforementioned operative applications. The existing instability for slow spinning momentum wheel, $\omega_{3_w} \in (0, 110\,\text{nd}]$ is due to the cross-coupling between the stored angular momentum along $\hat{\mathbf{b}}_3$ and the attitude dynamics around $\hat{\mathbf{b}}_1$ and $\hat{\mathbf{b}}_2$. Indeed, the slow spinning wheel is not sufficient to spin stabilise the system, but the cross-coupling makes the dynamics sensitive to perturbations perpendicular to the xy−plane and, thus, slightly unstable.

7. Modular and extended space structures in cislunar environment

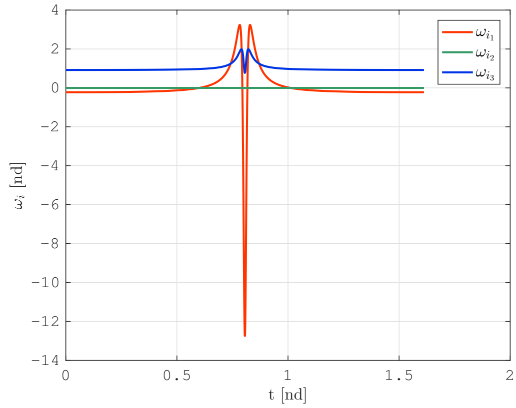
The analyses presented in the previous sections provides general results that can be exploited to drive the design of modular and extended space structures in cislunar environment. Many dedicated analyses can be carried out with this purpose, having in mind that the main driver while designing such a complex mission is related to a maximum reduction of the maintenance and station-keeping costs. Moreover, the progressive in-orbit assembly of the modular structure must be carefully planned in order to minimise risks and costs.

A first analysis is possible looking at a family of orbit-attitude periodic solutions with dual-spin attitude stabilisation, presented in fig. 9. In this case a family of EML2 Halo orbits is generated starting from a periodic solution with orbital period $T_{Halo_1} = 3.35\,\text{nd}$, maximum $\hat{\mathbf{z}}$ elongation $A_{z_{Max_1}} = 0.083\,\text{nd}$, angular rate and moment of inertia of the spinning wheel respectively $\omega_{3_w} = 100\,\text{nd}$ and $I_{3_w} = \frac{I_3}{100}$. The family is continued decreasing the spinning rate ω_{3_w} down to $-100\,\text{nd}$. At the beginning of the family, the orbit closest to the Moon, associated with the darker line in fig. 9a, and the dual-spin attitude stabilisation determine the convergence of the periodic solution with a fast quaternion dynamics influenced by the gravity gradient torque, associated to the sharp corner and the double inner loop in the quaternion subspace (darkest line in fig. 9b). Decreasing the spinning stabilisation, the convergence to a periodic solution is possible at a greater distance from the moon. The related attitude dynamics is influenced by a lower gyroscopic stiffness and a weaker gravity gradient torque, resulting in larger and smoother loop in fig. 9b. When the family reaches the point of momentum wheel with $\omega_{3_w} = -100\,\mathrm{nd}$ and, thus, a null gyroscopic stiffness, the converged periodic solution is at the largest distance from the Moon, where the librating attitude dynamics of the overall system is less affected by the gravity gradient.

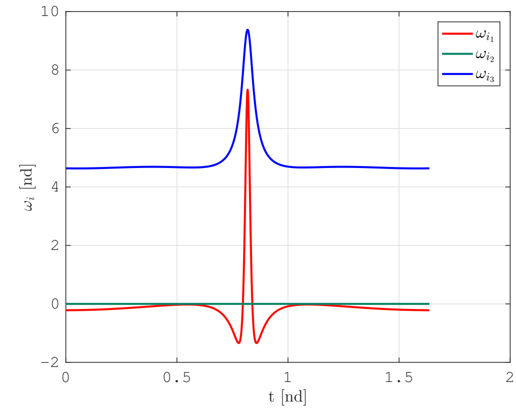
A similar family of orbit-attitude periodic dynamics can be obtained fixing the angular rate of the momentum wheel at a constant value and continuing the family along the inertia parameters of the system. Therefore, a certain periodic attitude motion, for a modular station with changing inertia properties, can be maintained by varying the attitude stabilisation level. Otherwise, for constant attitude stabilisation effort and different inertia parameters, the periodic motion can be achieved on a distinct orbit-attitude periodic solution.

The increased operational capabilities when attitude stabilisation techniques are exploited on extended and modular cislunar spacecraft are evident in fig. 10, where a spacecraft with $k_3 < 0$ is considered. According to classical attitude stability analyses,[10] pitch motion is stable only for positive inertia parameter k_3. In fact, for the currently analysed case with $k_3 = -0.2$, the resulting periodic motion in fig. 10a is extremely unstable with stability index, σ, equal to 21.5. Thus, an attitude stabilisation method is fundamental in order to correctly operate the given space system in a EML2 South NRO with period $T_{S-NRO} = 1.77\,\mathrm{nd}$ and maximum $\hat{\mathbf{z}}$ elongation $A_{z_{Max}} = -0.192\,\mathrm{nd}$. The performance of single-spin and dual-spin attitude stabilisation techniques are compared in figs. 10b and 10c, with details in the relative captions. Both solutions are effective in stabilising the spacecraft, determining a stability index respectively equal to $\sigma_{single} = 1.02$ for single-spin and $\sigma_{dual} = 2.16$ for dual-spin. The stabilised attitude evolution is completely transformed and the resulting dynamics has some analogies with the previously presented solution for a body with $k_3 = 0.2$. Therefore, for a modular space station that is progressively assembled, the situation in which a large attitude instability arises can be easily managed through a proper selection of the attitude stabilisation parameters.

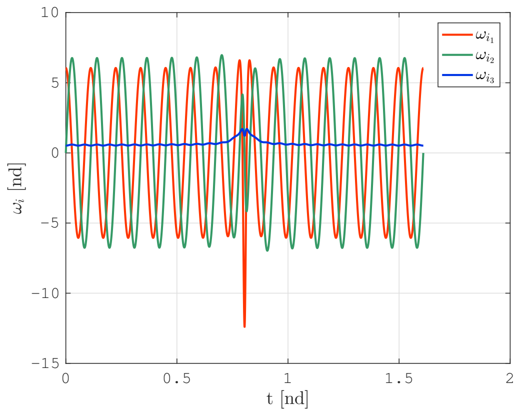
Many others analyses are possible with the presented dynamical model, such as the evaluation of attitude slew manoeuvres commanded by a non-constant spinning rate of the three momentum wheels or the robustness of a determined periodic solution to uncertain system parameters or perturbing phenomena modelisation. However, the main results presented in this section have already highlighted some preliminary drivers for the potential implementation of an extended and modular space station in cislunar space.



(a) Non-spin stabilised.



(b) Single-spin stabilised (1-spin).



(c) Dual-spin stabilised ($\omega_{3_w} = 5000\,\mathrm{nd}$, $I_{3_w} = \frac{I_3}{100}$).

Figure10.: Periodic orbit-attitude solutions on EML2 South NRO with $k_3 < 0$: $T_{S-NRO} = 1.77\,\mathrm{nd} = 7.68\,\mathrm{d}$, $A_{z_{Max}} = -0.192\,\mathrm{nd} = -7.41 \times 10^4\,\mathrm{km}$, $k_3 = -0.2$.

8. Final remarks

The orbit-attitude spin stabilized solutions presented in this paper laid a foundation for the total control of modular and extended space structures in cislunar environment. The attitude stabilisation techniques help the design of the considered space system, broadening the space of periodic orbit-attitude solutions that are stable enough to host an extended spacecraft with minimum control effort.

Moreover, the possibility to manage situations in which the inertia properties and the configurations of the space system change in time, is extremely important for a modular structure that will be assembled by means of many automated docking and undocking operations. In particular, because this capability is guaranteed by minor adjustment to the attitude stabilisation parameters.

Further studies are needed to extend the range of these preliminary results, and the investigation of an active control system with variable stored angular momentum is of interest. However, while designing this kind of space missions, the main driver that must be followed along the whole study, conception and implementation phase is the minimisation of maintenance and station-keeping costs. Thus, the attitude control technique should be as simple and efficient as possible. The attitude stabilisation methods presented in this paper, with single-spinning spacecrafts or constant speed spinning devices, showed a first positive result in this direction.

Even though the best orbit where a large space structure in the vicinity of the Moon can be effectively operated, is far to be completely defined, and the related coupled attitude dynamics has not yet been studied enough, this paper was intended to underline some relevant and essential conclusions in the field of dynamics and control of extended bodies in cislunar environment. Furthermore, the analyses were presented to emphasize the importance in studying the fully coupled orbit-attitude dynamics, together with the enhanced performances that can be achieved with plain stabilization methods, while designing a large and modular space structure in non-Keplerian orbits near the Moon.

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