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Estimation of Shape and Optical Parameters of Spinning Solar Sail Equipped with Reflectivity Control Devices

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Spinning solar sail equipped with Reflectivity Control Devices (RCD) is expected as an ideal spacecraft which does not require any fuel for orbit control and attitude control. In order to achieve more precise navigation guidance control it is necessary to estimate the shape and optical properties of the sail on orbit promptly and precisely. Therefore, we propose a estimation method of the shape and optical properties of the sail through estimating the model parameters of Generalized Spinning Sail Model (GSSM) which well expresses the attitude motion of spinning solar sail although the number of parameters is small. In the proposed method we achieved prompt and precise estimation by incorporating the nuation motion ignored by GSSM into the framework of GSSM.

Key Words: Spinning solar sail, Reflectivity control device, GSSM, Estimation

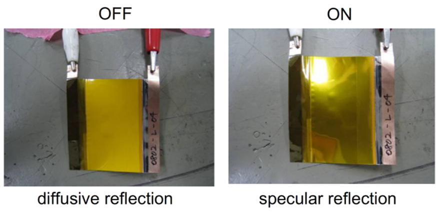

Fig. 1. Reflectivity Control Device(RCD)[1)]

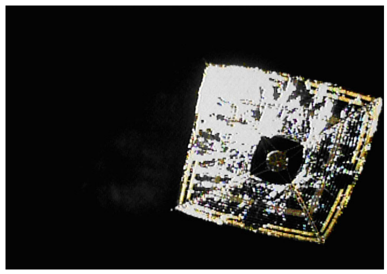

Fig. 2. JAXA's solar sail demonstration spacecraft IKAROS[2)]

Nomenclature

$\tilde{\alpha}$	:	azimuth angle relative to the solar direction
$\tilde{\delta}$	:	elevation angle relative to the solar direction
Ω	:	spin rate
ϕ	:	control input of RCD switching phase
I	:	moment of inertia

Subscripts

n	:	nutation motion

1. Introduction

Solar sails, which are accelerated by solar radiation pressure (SRP), do not require any fuel for propulsion. In particular, spinning solar sail is superior from the viewpoint of weight reduction and large size, and is expected to play a role in the field of deep space exploration in the future. On the other hand, the spinning solar sail has a disadvantage that a large amount of fuel is required for attitude control due to a large angular momentum due to spinning. As a solution for this disadvantage, Reflectivity Control Device (RCD)(Fig. 1), which can electrically generate torque, has been proposed.[1)] By using this RCD, it is possible to change the attitude of the solar sail only with electric energy, and in the future it will be possible to realize a spacecraft in which fuel is completely unnecessary in orbit control and attitude control.

To make space exploration missions with spin type solar sail more practical, it is necessary to achieve precise navigation, guidance and control, which requires a precise dynamics model including control by RCD. However, it has been revealed in IKAROS(Fig.2) mission that the attitude motion of spinning solar sail highly depends on SRP effects caused by slight deformation and variation of the optical properties of the sail surface.[2)] Importantly, these parameters are not determined until sail deployment is completed on orbit. Therefore, the parameters must be estimated from flight data promptly and precisely in order to predict and control the attitude.

However, little attention has been paid to the prompt and precise estimation of the parameters. The Generalized Sail Model (GSM) can precisely calculate the SRP torque exerted on the sail surface of an arbitrary shape, but there are thirty three parameters necessary for calculation and it is not suitable for estimation ,[3)] wheares Generalized Spinning Sail Model (GSSM) expresses the SRP effects by only three parameters, but it focuses only on the global motion that ignores the nutation motion, so the estimation is slow and not precise.[2)]

This paper proposes a prompt and precise estimation method of GSSM parameters by incorporating dynamics of nutaion motion, which is ignored in GSSM , into the framework of GSSM. We consider that the following benefits are obtained from this study.

- As the estimation is quick, estimation can be performed frequently, and it is also possible to cope with fluctuation of parameters due to spin rate change etc.
- The quick estimation (about several days) makes it possible to take longer time for orbital control (attitude control), and it is also advantageous immediately after launch (it can move to the orbital control sequence quickly)

This paper is organized as follows. Section 2 describes Attitude dynamics models as a prior study; GSM and GSSM. Section 3 proposes GSSM parameter estimation method. In Section 4, we constructed a simulator for estimation and estimate GSSM parameter on the simulator. Section 5 describes the conclusion and future work.

2. Attitude dynamics of spinning solar sail

2.1. Generalized Sail Model: GSM

Generalized Sail Model(GSM) is a model that precisely calculates the force and the torque of the SRP acting on the membrane surface of an arbitrary shape.[3] In GSM, the SRP torque $\boldsymbol{T}$ acting on the membrane surface is expressed by the following tensor calculation.

$$\boldsymbol{T} = P\left\{\boldsymbol{K}^2 \cdot \boldsymbol{r} + \boldsymbol{r} \cdot \boldsymbol{K}^3 \cdot \boldsymbol{r}\right\} \tag{1}$$

where P is the solar radiation pressure, and r is a unit vector in the direction from the sun to the sail surface. $\boldsymbol{K}^2$ and $\boldsymbol{K}^3$ are tensor components whose parameters are determined by the shape and optical properties of the sail surface.

2.2. Generalized Spinning Sail Model: GSSM

In GSSM, the following assumption is made for spin type solar sail rotating at the spin rate Ω around the z-axis of the aircraft fixed coordinate system and having inertia tensor $\boldsymbol{I} = \mathrm{diag}[I_T\ I_T\ I_S]$. Fig. 3 shows the definition of attitude angle representation

- The SRP torque acting on the film surface uses the average value for one rotation of the spin.
- The spin axis is close enough to the sun direction and the ecliptic plane($|\tilde{\alpha}|$, $|\tilde{\delta}|$, $|\delta| \ll 1$)。
- Ignore the nutation motion (nutation) of the spin axis.

Under the above assumption, the Equations and dynamics of the SRP torque $\boldsymbol{T}$ exerting on the spinning solar sail are derived.

$$\boldsymbol{T} = \begin{bmatrix} A & -B & 0 \\ B & A & 0 \\ 0 & 0 & C \end{bmatrix} \begin{bmatrix} \tilde{\alpha} \\ \tilde{\delta} \\ 1 \end{bmatrix} \tag{2}$$

$$\frac{d}{dt}\begin{bmatrix} \tilde{\alpha} \\ \tilde{\delta} \\ \Omega \end{bmatrix} = \frac{1}{I_S \Omega}\begin{bmatrix} A & -B & 0 \\ B & A & 0 \\ 0 & 0 & C \end{bmatrix} \begin{bmatrix} \tilde{\alpha} \\ \tilde{\delta} \\ \Omega \end{bmatrix} - \begin{bmatrix} \dot{\alpha}_s \\ \dot{\delta}_s \\ 0 \end{bmatrix} \tag{3}$$

where A, B, C are parameters determined by slight deformation and optical properties of the sail surface. In addition, in the previous work,[4] similar assumptions were introduced to derive the increment of SRP torque, $\varDelta\boldsymbol{T}$, generated when there is RCD control input.

$$\boldsymbol{T} = \begin{bmatrix} A & -B & 0 \\ B & A & 0 \\ 0 & 0 & C \end{bmatrix} \begin{bmatrix} \tilde{\alpha} \\ \tilde{\delta} \\ 1 \end{bmatrix}. \tag{4}$$

$$\varDelta\boldsymbol{T} = \begin{bmatrix} \varDelta A & -\varDelta B & 0 \\ \varDelta B & \varDelta A & 0 \\ -\varDelta D\sin\phi - \varDelta E\cos\phi & \varDelta D\cos\phi - \varDelta E\sin\phi & \varDelta C \end{bmatrix} \begin{bmatrix} \tilde{\alpha} \\ \tilde{\delta} \\ 1 \end{bmatrix} + \begin{bmatrix} \sin\phi & \cos\phi & 0 \\ -\cos\phi & \sin\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ 0 \end{bmatrix} \tag{5}$$

where $\varDelta A$, $\varDelta B$, $\varDelta C$, $\varDelta D$, $\varDelta E$, H_1 *and* H_2 are parameters determined by the shape and optical properties of the RCD, and ϕ is a control parameter representing the phase angle at which the RCD is turned ON. Also, when all the RCDs are turned ON, the following torque is generated.

$$\varDelta\boldsymbol{T} = \begin{bmatrix} 2\varDelta A & -2\varDelta B & 0 \\ 2\varDelta B & 2\varDelta A & 0 \\ 0 & 0 & 2\varDelta C \end{bmatrix} \begin{bmatrix} \tilde{\alpha} \\ \tilde{\delta} \\ 1 \end{bmatrix} \tag{6}$$

3. Proposed estimation method

3.1. Parameters to be estimated

In estimating the parameters of the GSSM, we will reorganize the estimation target. The target to be estimated is the parameter A, B, C, $\varDelta A$, $\varDelta B$, $\varDelta C$, $\varDelta D$, $\varDelta E$, H_1 and H_2 appearing in the following torque equation

$$\boldsymbol{T} = \begin{bmatrix} A & -B & 0 \\ B & A & 0 \\ 0 & 0 & C \end{bmatrix} \begin{bmatrix} \tilde{\alpha} \\ \tilde{\delta} \\ 1 \end{bmatrix}. \tag{7}$$

$$\varDelta\boldsymbol{T} = \begin{bmatrix} \varDelta A & -\varDelta B & 0 \\ \varDelta B & \varDelta A & 0 \\ -\varDelta D\sin\phi - \varDelta E\cos\phi & \varDelta D\cos\phi - \varDelta E\sin\phi & \varDelta C \end{bmatrix} \begin{bmatrix} \tilde{\alpha} \\ \tilde{\delta} \\ 1 \end{bmatrix} + \begin{bmatrix} \sin\phi & \cos\phi & 0 \\ -\cos\phi & \sin\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ 0 \end{bmatrix}. \tag{8}$$

Since all parameters appear in the torque equation, we can observe the angular velocity ω_x, ω_y, Ω or attitude angle $\tilde{\alpha}$, $\tilde{\delta}$ and estimate the parameters from the time series data.

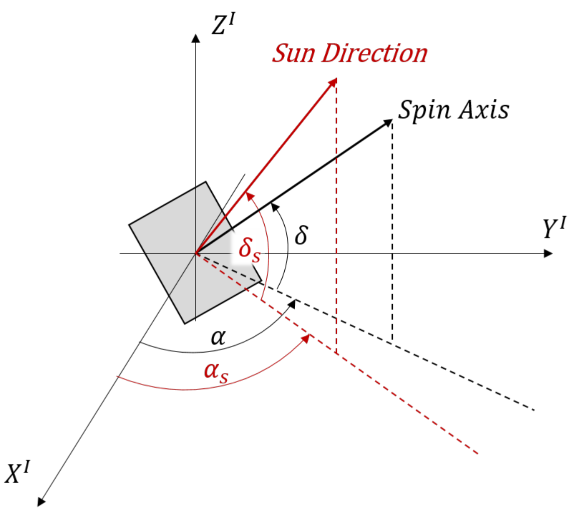

Fig. 3. Attitude angle representation relative to inertial coordinate system

This paper first proposes a method to estimate parameter A, B and C without RCD control, and estimates parameter $\varDelta A$, $\varDelta B$, $\varDelta C$, $\varDelta D$, $\varDelta E$, H_1 and H_2 with RCD control by using the same method.

3.2. Estimation method for parameters without RCD control

In this study, dynamics is modeled by adding nutation motion to GSSM. There, we set the following assumptions.

Assumption in modeling nutation motion

$\tilde{\alpha}, \tilde{\delta}$ are expressed by the sum of $\alpha_n,\ \delta_n$ extracting only the nutaion motion and $\overline{\alpha},\ \overline{\delta}$ extracting only the global motion.

$$\tilde{\alpha} = \alpha_n + \overline{\alpha},\ \tilde{\delta} = \delta_n + \overline{\delta} \quad (9)$$

Under this assumption, the dynamics of the attitude angle can be divided into the following expressions.

$$\frac{d}{dt}\begin{bmatrix}\alpha_n\\ \delta_n\end{bmatrix} = \begin{bmatrix}0 & -\frac{I_S}{I_T}\Omega\\ \frac{I_S}{I_T}\Omega & 0\end{bmatrix}\begin{bmatrix}\alpha_n\\ \delta_n\end{bmatrix}. \quad (10)$$

$$\frac{d}{dt}\begin{bmatrix}\overline{\alpha}\\ \overline{\delta}\end{bmatrix} = \frac{1}{I_S\Omega}\begin{bmatrix}A & -B\\ B & A\end{bmatrix}\begin{bmatrix}\overline{\alpha}\\ \overline{\delta}\end{bmatrix} - \begin{bmatrix}\dot{\alpha}_s\\ \dot{\delta}_s\end{bmatrix}. \quad (11)$$

Also, the dynamics of the spin rate Ω is the same as GSSM.

$$\dot{\Omega} = \frac{C}{I_S}. \quad (12)$$

With this division, considering the nutation motion, the global motion dominated by the parameters $A,\ B$ and C don't be buried in the nutation motion.

3.2.1. Construction of state space model

This section constructs a state space model that considers nutation motion for estimation. Defining the state vector as $\boldsymbol{x} = [\alpha_n,\ \delta_n,\ \overline{\alpha},\ \overline{\delta},\ \Omega]^{\mathrm{T}}$ and parameter vector as $\boldsymbol{\theta} = [A,\ B,\ C]^{\mathrm{T}}$, the following equation is derived.

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{f}(\boldsymbol{x},\ \boldsymbol{\theta}) + \boldsymbol{v} = \begin{bmatrix}-\frac{I_S}{I_T}\Omega\delta_n\\ \frac{I_S}{I_T}\Omega\alpha_n\\ \frac{A}{I_S\Omega}\overline{\alpha} - \frac{B}{I_S\Omega}\overline{\delta}\\ \frac{B}{I_S\Omega}\overline{\alpha} + \frac{A}{I_S\Omega}\overline{\delta}\\ \frac{C}{I_S}\end{bmatrix} + \boldsymbol{v} \quad (13)$$

where $\boldsymbol{v}$ is the process noise.

Also, when choosing attitude angle and spin rate as the observation vector $\boldsymbol{y}$, the following observation equation is derived.

$$\boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x}) + \boldsymbol{w} = \begin{bmatrix}\alpha_n + \overline{\alpha}\\ \delta_n + \overline{\delta}\\ \Omega\end{bmatrix} + \boldsymbol{w} \quad (14)$$

where $\boldsymbol{w}$ is the measurement noise. From the above, the problem of estimating parameters $A,\ B$ and C is replaced by the problem of optimizing $\boldsymbol{\theta}$ based on the state space model from given observation data.

3.2.2. Construction of Self-Organizing State Space Model

In this study, we use a Self-Organizing State Space Model(SOSS model) that simultaneously obtains estimates of parameters and state vector.[5] The SOSS model defines the extended state vector $\boldsymbol{z}$ which is a combination of the state vector $\boldsymbol{x}$ and the parameter vector $\boldsymbol{\theta}$.

$$\boldsymbol{z} = [\boldsymbol{x},\ \boldsymbol{\theta}]^{\mathrm{T}} = [\alpha_n,\ \delta_n,\ \overline{\alpha},\ \overline{\delta},\ \Omega,\ A,\ B,\ C]^{\mathrm{T}}. \quad (15)$$

Then, the extended state equation and the extended observation equation are defined as follows.

$$\frac{d\boldsymbol{z}}{dt} = \tilde{\boldsymbol{f}}(\boldsymbol{z}) + \begin{bmatrix}\boldsymbol{v}\\ \boldsymbol{0}\end{bmatrix} = \begin{bmatrix}-\frac{I_S}{I_T}\Omega\delta_n\\ \frac{I_S}{I_T}\Omega\alpha_n\\ \frac{A}{I_S\Omega}\overline{\alpha} - \frac{B}{I_S\Omega}\overline{\delta}\\ \frac{B}{I_S\Omega}\overline{\alpha} + \frac{A}{I_S\Omega}\overline{\delta}\\ \frac{C}{I_S}\\ 0\\ 0\\ 0\end{bmatrix} + \begin{bmatrix}\boldsymbol{v}\\ 0\\ 0\\ 0\end{bmatrix}. \quad (16)$$

$$\boldsymbol{y} = \tilde{\boldsymbol{h}}(\boldsymbol{z}) + \boldsymbol{w} = \begin{bmatrix}\alpha_n + \overline{\alpha}\\ \delta_n + \overline{\delta}\\ \Omega\end{bmatrix} + \boldsymbol{w}. \quad (17)$$

Thus, in the SOSS model, it is possible to adapt usual filter theory such as the Kalman filter to the parameter estimation problem by extending the state space model. As a result, estimated values of the parameters $A,\ B,\ C$ are obtained.

3.3. Estimation method for parameters with RCD control

To estimate parameters related to RCD control $\varDelta A$, $\varDelta B$, $\varDelta C$, $\varDelta D$, $\varDelta E$, H_1 and H_2, it is necessary to give a control input to the RCD and generate a torque increment. In this study, we consider two control input strategies and estimate parameters with RCD control.

Strategy 1. Turn on all RCD

With this strategy, the following torque is generated in the sail.

$$\boldsymbol{T} + \varDelta\boldsymbol{T} = \begin{bmatrix}A + 2\varDelta A & -(B + 2\varDelta B) & 0\\ B + 2\varDelta B & A + 2\varDelta A & 0\\ 0 & 0 & C + 2\varDelta C\end{bmatrix}\begin{bmatrix}\tilde{\alpha}\\ \tilde{\delta}\\ 1\end{bmatrix} \quad (18)$$

This equation is equal to the expression of the torque acting on the uncontrolled sail replacing $A \rightarrow A + 2\varDelta A$, $B \rightarrow B + 2\varDelta B$, $C \rightarrow C + 2\varDelta C$. That is, the estimation method in the previous section can be applied. By taking the difference between the estimated parameters $A + 2\varDelta A$, $B + 2\varDelta B$, $C + 2\varDelta C$ and already estimated $A,\ B,\ C$, we can get an estimate of the parameters $\varDelta A$, $\varDelta B$, $\varDelta C$.

Strategy 2. Turn on RCD for one side

With this strategy, the torque as shown in Eq.(5) is additionally generated in the sail. As in the previous section, by constructing the SOSS model and adapting the filter theory, the remaining parameter $\varDelta D$, $\varDelta E$, H_1, H_2 is obtained.

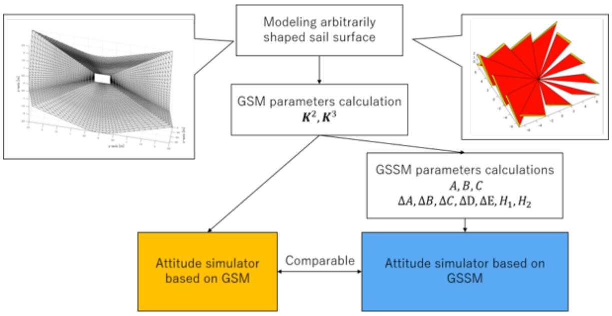

Fig. 4. Outline of the attitude simulator

4. Numerical Simulation

4.1. Precise model attitude simulator

To verify the proposed method, we created an attitude motion simulator of spinnig solar sail(outlined in Fig. 4). Although the estimation target in this research is a parameter of the GSSM, nutation motion can not occur in the simulator based on only GSSM framework, and the parameter estimation result becomes an ideal value. Therefore, we constructed a simulator based on the precise model GSM, and compared the parameter estimate with the true parameter value. In this case, we derive the following equation which can calculate the true value of GSSM from the GSM parameters.

$$
\begin{aligned}
A &= \frac{1}{2}P\left(K_{11}^2 + K_{22}^2 - K_{113}^3 - K_{223}^3 - K_{311}^3 - K_{322}^3\right).\\
B &= \frac{1}{2}P\left(K_{12}^2 - K_{21}^2 - K_{123}^3 + K_{213}^3 + K_{312}^3 - K_{321}^3\right).\\
C &= P\left(-K_{33}^2 + K_{333}^3\right).\\
\varDelta A &= \frac{1}{4}P\left(K_{11}^2 + K_{22}^2 - K_{113}^3 - K_{223}^3 - K_{311}^3 - K_{322}^3\right).\\
\varDelta B &= \frac{1}{4}P\left(K_{12}^2 - K_{21}^2 - K_{123}^3 + K_{213}^3 + K_{312}^3 - K_{321}^3\right).\\
\varDelta C &= \frac{1}{2}P\left(-K_{33}^2 + K_{333}^3\right).\\
\varDelta D &= \frac{1}{\pi}P\left(K_{13}^2 - K_{133}^3 - K_{331}^3\right).\\
\varDelta E &= \frac{1}{\pi}P\left(K_{23}^2 - K_{233}^3 - K_{332}^3\right).\\
H_1 &= \frac{1}{\pi}P\left(K_{31}^2 - K_{313}^3\right).\\
H_2 &= \frac{1}{\pi}P\left(K_{32}^2 - K_{323}^3\right).
\end{aligned}
\tag{19}
$$

4.2. Estimation of parameters without RCD control by EKF

We applied Extended Kalman Filter(EKF) to the SOSS model defined in the previous section, where the parameters A, B, C were estimated simultaneously with the attitude angle and the spin rate. The observation period was 12 hours and the initial angular velocity was $\boldsymbol{\omega} = [0.001,\ 0,\ 1.5]^{\mathrm{T}}$ [rpm] (Other specifications are given in Appendix).

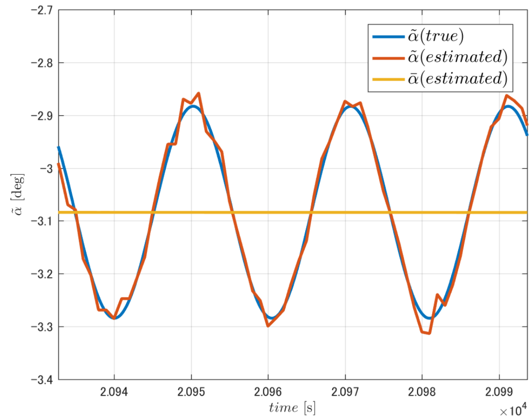

Fig. 5. Enlarged view of estimated results of $\tilde{\alpha}$ and $\bar{\alpha}$ by the proposed method

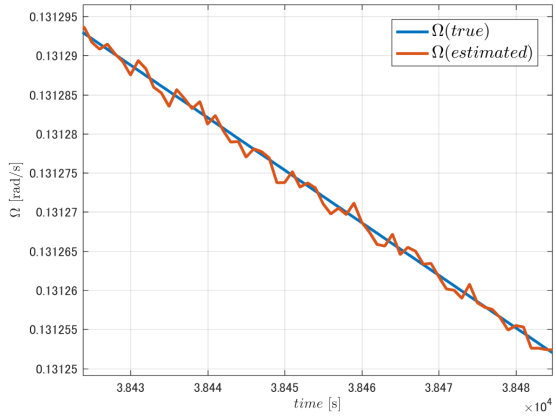

Fig. 6. Enlarged view of estimation result of Ω by proposed method

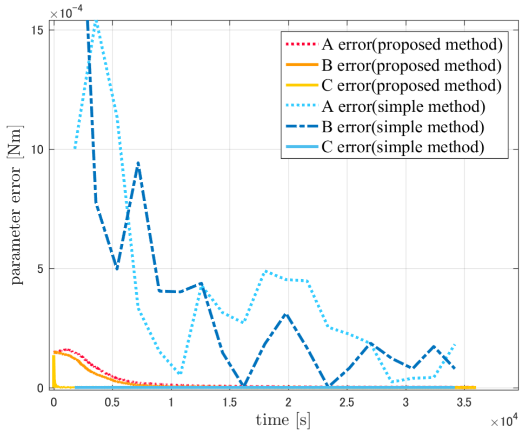

Fig. 7. Estimation results of GSSM parameters A, B, C by the proposed method and simple estimation method without considering the nutation motion

From Fig. 5 and Fig. 6, we can confirm that state vactor values such as attitude angle and spin rate can be estimated by applying EKF.

Simultaneously with these state vector values, the result of estimating the parameters A, B, C is shown in Fig. 7. For comparison, the results estimated using a simple estimation method focusing only on global motion without considering the nutation motion are also shown. Comparing the proposed

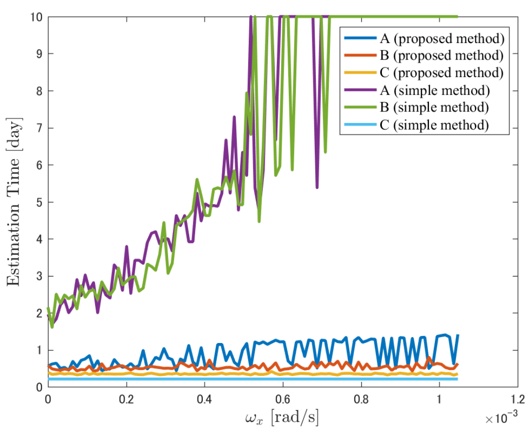

Fig. 8. Convergence time of estimated value for the magnitude of nutation motion, i.e. initial angular velocity ω_x

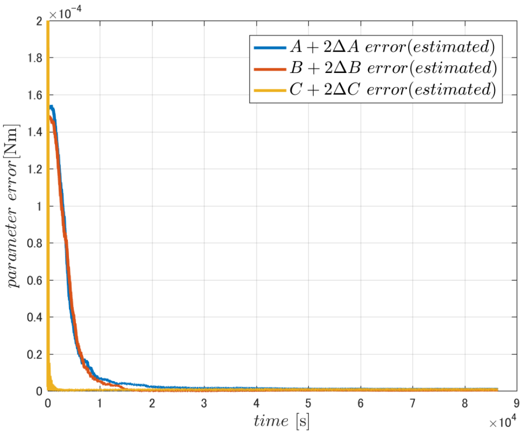

Fig. 9. Estimated results of GSSM parameters $A+2\mathit{\Delta}A$, $B+2\mathit{\Delta}B$, $C+2\mathit{\Delta}C$ by the proposed method

method with the simple estimation method without considering the nutation motion, the convergence of the parameters A and B is particularly fast in the proposed method. This is due to the difference whether or not considering the nutation motion. The effect of the magnitude of nutaion motion on the convergence time of the estimate is shown in Fig. 8.

4.3. Estimation of parameters with RCD control by EKF

Strategy 1. Turn on all RCD

Applying EKF to the same SOSS model with the Strategy 1 and estimating the parameters $A + 2\mathit{\Delta}A$, $B + 2\mathit{\Delta}B$, $C + 2\mathit{\Delta}C$ at the same time as the attitude angle and spin rate is shown in the Fig. 9. The observation period is 12 hours and the initial angular velocity is $\boldsymbol{\omega} = [0.001,\ 0,\ 1.5]^{\mathrm{T}}$ [rpm].

Strategy 2. Turn on RCD for one side

Finally, estimate the parameters $\mathit{\Delta}D$, $\mathit{\Delta}E$, H_1, H_2. We applied EKF to the SOSS model with the Strategy 2 to estimate the parameters $\mathit{\Delta}D$, $\mathit{\Delta}E$, H_1, H_2 at the same time as the attitude angle and spin rate The result is shown in Fig. 10. The observation period is 12 hours, the initial angular velocity is $\boldsymbol{\omega} = [0.001,\ 0,\ 1.5]^{\mathrm{T}}$ [rpm], the control input is $\phi = 0$.

$\mathit{\Delta}D$, $\mathit{\Delta}E$ did not converge, and H_1, H_2 resulted in the estimated value converging to almost true value.

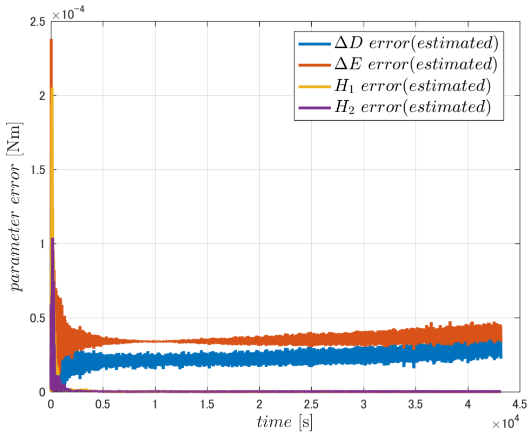

Fig. 10. Estimated results of GSSM parameters $\mathit{\Delta}D$, $\mathit{\Delta}E$, H_1, H_2 by the proposed method

5. Conclusion

Parameter estimation of GSSM was performed using a precision model attitude simulator corresponding to control input of RCD. It can be said that an estimation method with a short convergence time of estimate could be established by SOSS model considering nutation motion. Although it was able to estimate many GSSM parameters, it turned out that some parameters can not be estimated unless some control input is devised.

As a future work, we would like to conduct active sensing research that optimizes control inputs and estimates parameters.

Appendix

Simulation for parameter estimation was performed using the parameters shown in Table. 1 and the sail model shown in Fig. 11.

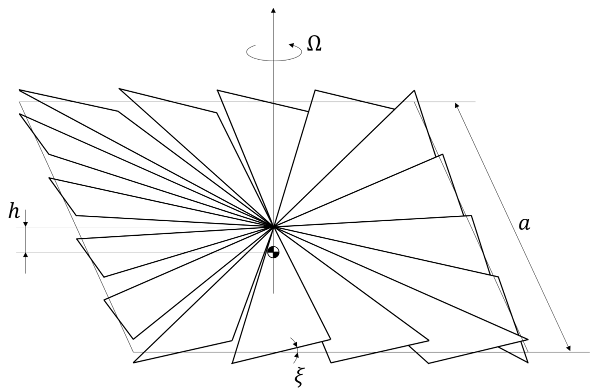

Fig. 11. Sail model

Table 1. Simulation data on parameter estimation

Inertia tensor $\boldsymbol{I}$	diag[400 400 800] [kgm^2]
Division number	120
The length of one side of the sail a	14 [m]
sail-center of gravity distance h	0.04 [m]
Twist angle of sail ξ	-1.0 [deg]
Warping angle of film surface η	-1.0 [deg]
Total reflectance ρ	0.84
Specularity s	0.857
RCD width r	0.3 [m]
Total reflectance of RCD(ON)	0.58
Specularity of RCD(ON)	0.6552
Total reflectance of RCD(OFF)	0.55
Specularity of RCD(OFF)	0.1455
Initial attitude angle $(\tilde{\alpha},\ \tilde{\delta})$	$(-3,\ -3)$ [deg]
Change rate of sun direction $(\dot{\alpha}_s,\ \dot{\delta}_s)$	$(\frac{2\pi}{60\times60\times24\times365},\ 0)$ [rad]
SRP P	4.58×10^{-6} [Pa]
Gyro sensor measurement noise(1σ)	1.0×10^{-6} [rad/s]
Sun sensor measurement noise(3σ)	0.05 [deg]
Observation frequency	1 [Hz]

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