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Indirect Optimization of End-of-Life Disposal for Galileo Constellation Using Low Thrust Propulsion

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In this work, the end-of-life disposal of satellites in the Galileo constellation using low thrust propulsion is studied. Indirect optimization methods are employed to design transfer maneuvers to remove the satellite from its original operational orbit into previously computed orbits leading to its natural re-entry within 100 years due to lunisolar perturbation effects. The dynamics are formulated using the modified equinoctial elements, which allow expressing the boundary conditions in a simple way at the cost of more complex equations compared to the use of Cartesian coordinates. A special focus is placed in defining an efficient and robust algorithm for solving the two point boundary value problem arising from the first order optimality conditions, including the integration of the analytically-derived variational equations to obtain the State Transition Matrix, and the accurate detection of thrust-switching events. The numerical results obtained for several test cases show the practical feasibility of this end-of-life disposal approach at thrust levels compatible with electric thrusters likely to be used by the next generation of Galileo satellites.

Key Words: Trajectory Optimization, Space Debris, Low Thrust, Modified Equinoctial Elements

Nomenclature

μ	:	Earth's gravitational parameter
a	:	semi-major axis
e	:	eccentricity
ω	:	argument of perigee
Ω	:	right ascension of the ascending node
i	:	orbital inclination
θ	:	true anomaly
p	:	semilatus rectum
e_x	:	modified orbital element
e_y	:	modified orbital element
h_x	:	modified orbital element
h_x	:	modified orbital element
L	:	true longitude
t	:	time
λ_+	:	costate associated to +
$\mathbf{s}$	:	state vector
$\boldsymbol{\lambda}$	:	costates vector
m	:	mass of the spacecraft
$T_{\max}$	:	maximum available thrust
I_{sp}	:	specific impulse
g_0	:	gravity acceleration at sea level
Δv	:	total impulse
f	:	thrust throttling parameter
$\boldsymbol{\alpha}$	:	thrust pointing unity vector
J	:	objective (cost) function
$\mathcal{L}$	:	Lagrangian
$\mathcal{H}$	:	Hamiltonian
Subscripts		
i	:	initial
f	:	final

1. Introduction

The commercial use of space is expected to rapidly grow in the coming years, including satellite constellations for diverse applications such as navigation, communications, Earth imaging or resource mapping. Given the increasing number of objects launched into orbit, a sustained, secure and profitable exploitation of space requires end-of-life (EoL) disposal strategies to be applied. In the case of Medium Earth Orbit (MEO) regions two options are available: 1) move the satellites into graveyard orbits, 2) insert them into Earth re-entry paths. The latter shall be preferred as it physically removes the satellite, however it is generally associated with high Δv and re-entry times (~100 m/s for a re-entry in 100 years). With current chemical propulsion systems this solution is not practical, but the scenario will change when fully-electric platforms will be utilized. This can be the case of the next generation satellites in the Galileo constellation.[1,18)]

The proposed low thrust maneuver aims to move the satellite from its original orbit into a new one leading to its re-entry, but the particular positions at which the transfer starts and ends need not to be fixed. Therefore, this optimal control problem is more naturally expressed using an orbital elements-based formulation, such as the modified equinoctial elements.[6)] However, this choice introduces additional complications compared to the classic Cartesian coordinates when solving the problem using the indirect method. The well-known result of the primer vector is no longer valid, and the straightforward analytical derivation of the variational equations, required to efficiently solve the associated two-point boundary value problem, becomes too cumbersome to be practical (even with symbolic manipulators). A careful treatment of the state and costate equations is then needed to derive variational equations and to develop an efficient and robust solver.

In this work, the design of low thrust EoL disposal trajectories for satellites in the Galileo constellation is studied, using the modified equinoctial elements to model dynamics and the indirect method to solve the associated optimal control problem. A set of previously computed orbits that exploit the lunisolar perturbation forces in the MEO region to naturally achieve re-entry times within 100 years[1,2] are considered as candidate arrival conditions, while the departure ones are given by the actual orbits of the Galileo constellation. The use of the modified equinoctial elements allows us to express the boundary conditions in a simple way, where all the initial and final elements are fixed except for the true longitude (determining the particular points in the initial and final orbits where the transfer maneuver starts and ends respectively). This easier treatment of the boundary conditions comes at the cost of more complex expressions for the two-point boundary value problem (TPBVP) arising from the first order optimality conditions in the indirect method. A careful study of these equations and their behavior is then performed to implement an efficient and robust solver for the TPBVP following the structure in 7), including the integration of the analytically-derived variational equations to obtain the State Transition Matrix and the accurate detection and treatment of thruster-switching events. Finally, a representative set of test cases for different initial and final orbits is studied using this solver.

2. Problem Statement

2.1. Galileo disposal

Galileo is the Global Navigation Satellite System currently being developed by the European Union and the European Space Agency.[17] Once completed, it will consist of 24 satellites plus 6 spares, located in three MEO planes at an altitude of 23,222 km and an orbital inclination of 56 deg.

Recent works by Armellin et al.[1,2] have studied the possibility of leveraging lunisolar perturbation effects to achieve the EoL disposal of Galileo satellites through a single impulsive maneuver placing the satellite in a new orbit that leads to its natural re-entry within a 100 years timespan. Although effective, this strategy has the drawback of requiring Δv levels impractically high for the impulsive thrusters currently used in Galileo. However, this limitation may be removed in the future generations of Galileo satellites, which are expected to feature electric thrusters[18] such as the T6 from QinetiQ.[3–5]

This work extends the previous Galileo disposal studies by replacing the impulsive maneuver with a minimum-fuel low thrust transfer between the operational and disposal orbits, for values of $T_{\max}$ and I_{sp} typical of the T6 ion engine.[4,5] Note that only the departure and arrival orbits need to be imposed, not the actual position of the spacecraft on them. For simplicity, the departure position and mass will be considered fixed from now on, whereas their final values are set free. Because the duration of the maneuver is very short compared to the re-entry time (tens of days versus a hundred years), a model including only Earth's central attraction and the thrust acceleration is used.

2.2. Equations of motion

The dynamics are formulated using the modified equinoctial elements (MEEs).[6,8–10] Compared to Cartesian coordinates, these have the key advantage of allowing to express the orbit-to-orbit boundary conditions as fixed values for some elements of the initial and final states, rather than as nonlinear functions of them. For a direct orbit, the MEEs can be related with the classical orbital elements as follows

$$\begin{aligned} p &= a\left(1-e^2\right) \\ e_x &= e\cos(\omega+\Omega) \\ e_y &= e\sin(\omega+\Omega) \\ h_x &= \tan(i/2)\cos(\Omega) \\ h_y &= \tan(i/2)\sin(\Omega) \\ L &= \omega+\Omega+\theta \end{aligned} \tag{1}$$

It is straightforward to check that p, e_x, e_y, h_x and h_y define the orbit, whereas L indicates the position of the spacecraft inside it. By introducing

$$\mathbf{s} = \begin{bmatrix} p & e_x & e_y & h_x & h_y & L \end{bmatrix}^\top, \quad \mathbf{x} = \begin{bmatrix} \mathbf{s} & m \end{bmatrix},$$

as well as the control vector

$$\mathbf{u} = \begin{bmatrix} f & \boldsymbol{\alpha} \end{bmatrix}^\top$$

where $f \in [0,1]$ is the thrust throttling parameter and $\boldsymbol{\alpha}$ the thrust pointing unitary vector, the equations of motion, including the mass equation, take the form

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x},\mathbf{u},t) := \begin{bmatrix} \mathbf{B} f \dfrac{T_{\max}}{m}\boldsymbol{\alpha} + \mathbf{A} \\ -\dfrac{T_{\max}}{c} f \end{bmatrix} \tag{2}$$

where

$$c = I_{\text{sp}} g_0$$

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & W^2\sqrt{\mu/p^3} \end{bmatrix}^\top$$

$$\mathbf{B} = \sqrt{\frac{p}{\mu}} \begin{bmatrix} 0 & \frac{2p}{W} & 0 \\ \sin L & \frac{1}{W}\left[(W+1)\cos L + e_x\right] & -\frac{e_y}{W}K \\ -\cos L & \frac{1}{W}\left[(W+1)\sin L + e_y\right] & \frac{e_x}{W}K \\ 0 & 0 & \frac{s^2\cos L}{2W} \\ 0 & 0 & \frac{s^2\sin L}{2W} \\ 0 & 0 & \frac{K}{W} \end{bmatrix}$$

with

$$\begin{aligned} W &= 1 + e_x\cos L + e_y\sin L \\ K &= h_x\sin L - h_y\cos L \\ s^2 &= 1 + h_x^2 + h_y^2 \end{aligned}$$

2.3. Indirect optimization method

Indirect optimization methods are based on deriving the first order optimality conditions for the problem using the calculus of variations. This leads to a TPBVP in the state and its associated costates, which normally has to be solved using numerical methods.

Although the aim of the study is to design minimum fuel transfers, this task cannot be tackled directly. First of all, a minimum-time solution is needed to determine the feasible region for the time of flight. The objective function for this minimum-time problem takes the form

$$\hat{J} = \int_{t_i}^{t_f} 1\,\mathrm{d}t = \int_{t_i}^{t_f} \hat{\mathcal{L}}\,\mathrm{d}t. \tag{3}$$

Secondly, the bang-bang structure of minimum-fuel solutions introduces discontinuities that severely affect the performance of numerical solvers. This issue can be addressed by following a homotopic approach.[7,11] To this end, an objective function J dependent on an homotopy parameter ε is defined as follows

$$J = \frac{T_{\max}}{c} \int_{t_i}^{t_f} [f - \varepsilon f (1 - f)]\, \mathrm{d}t = \int_{t_i}^{t_f} \mathcal{L}\, \mathrm{d}t\,,\ \varepsilon \in [0, 1] \tag{4}$$

Note that the two limit values 0 and 1 of the homotopy parameter correspond, respectively, to the minimum-fuel and minimum-energy. Then, the homotopic approach would consist on solving the (easier) minimum-energy problem first, and then performing a continuation in ε until the minimum-fuel solution is reached.

The Hamiltonian for the minimum-energy/fuel problems is[12]

$$\mathcal{H} = \boldsymbol{\lambda} \cdot \mathbf{F} + \mathcal{L} \tag{5}$$

where $\boldsymbol{\lambda} = [\boldsymbol{\lambda}_\mathbf{s}\, \lambda_m\,]$ is the vector of costates, with $\boldsymbol{\lambda}_\mathbf{s} = [\,\lambda_p\, \lambda_{e_x}\, \lambda_{e_y}\, \lambda_{h_x}\, \lambda_{h_y}\, \lambda_L\,]$. Note that the Hamiltonian for the minimum-time problem will have the same structure, simply replacing $\mathcal{L}$ with $\hat{\mathcal{L}}$. The differential equations governing the evolution of the costates can be obtained by imposing the first order optimality conditions derived from the calculus of variations,[12] leading to:

$$\dot{\boldsymbol{\lambda}} = -\frac{\partial \mathcal{H}}{\partial \mathbf{x}} \tag{6}$$

The boundary conditions for the costate depend on the boundary conditions imposed to the state. In particular, the initial (or final) value of a costate component will be zero if the corresponding initial (or final) state is free, and an unknown parameter otherwise.[12] Since only the final true longitude and mass are free, the costates associated to L and m will be zero at t_f

$$\lambda_L\left(t_f\right) = 0\,, \quad \lambda_m\left(t_f\right) = 0\,,$$

whereas the rest will be unknowns parameters of the TPBVP. Note that, same as with the state, the use of MEEs allows to express the boundary conditions in a simple way.

Control variables f and $\boldsymbol{\alpha}$ can be expressed as functions of the state and costate by applying the Pontryagin Maximum Principle,[13] which states that the optimal control for a given optimal trajectory is the one that leads to an extreme value (in this case a minimum) of $\mathcal{H}$ over the set of admissible controls. Gathering the terms involving control variables in Eq. (5) one reaches

$$\mathcal{H} = \boldsymbol{\lambda}_\mathbf{s} \cdot \mathbf{A} + f \frac{T_{\mathbf{max}}}{c} \left[\frac{c}{m} \boldsymbol{\lambda}_\mathbf{s} \mathbf{B} \boldsymbol{\alpha} - \lambda_m + 1 - \varepsilon (1 - f)\right] \tag{7}$$

Because $f T_{\mathbf{max}}/c$ is always semi-positive, minimizing the Hamiltonian with respect to $\boldsymbol{\alpha}$ amounts to choosing its value so that the first term inside the brackets is as small as possible. This leads to:

$$\boldsymbol{\alpha}^* = -\frac{\mathbf{B}^\top \cdot \boldsymbol{\lambda}_\mathbf{s}}{||\mathbf{B}^\top \cdot \boldsymbol{\lambda}_\mathbf{s}||} \tag{8}$$

This expression for the thrust orientation is notably more complex than the primer vector in Cartesian coordinates,[14] and constitutes one of the main drawbacks of using the MEEs. Substituting $\boldsymbol{\alpha}^*$ back into Eq. (7) yields

$$\mathcal{H} = \boldsymbol{\lambda}_\mathbf{s} \cdot \mathbf{A} + f \frac{T_{\mathbf{max}}}{c} [S - \varepsilon + \varepsilon f] \tag{9}$$

where a switching function S has been introduced as

$$S = -\frac{c}{m} \frac{\boldsymbol{\lambda}_\mathbf{s}^\top \mathbf{B} \mathbf{B}^\top \boldsymbol{\lambda}_\mathbf{s}}{||\mathbf{B}^\top \cdot \boldsymbol{\lambda}_\mathbf{s}||} - \lambda_m + 1 \tag{10}$$

It is now straightforward to analytically find the minimum of $\mathcal{H}$ with respect to f by deriving and equating to zero, which leads to $f = (\varepsilon - S)/2\varepsilon$. However, because f is limited to the range [0, 1], the optimal throttling takes the form of a piecewise function

$$f^* = \begin{cases} 0 & \text{for } S > \varepsilon \\ (\varepsilon - S)/2\varepsilon & \text{for } -\varepsilon \leq S \leq \varepsilon \\ 1 & \text{for } S < -\varepsilon \end{cases} \tag{11}$$

where the value of the switching function allows determining the operation mode for the thruster: off, intermediate, or full on. Note that in the minimum-fuel case ε is zero and the intermediate region vanishes, so leading to a bang-bang profile.

Regarding the minimum-time problem, the expression for $\boldsymbol{\alpha}^*$ in Eq. (8) still holds, and the Hamiltonian takes the form

$$\hat{\mathcal{H}} = \boldsymbol{\lambda}_\mathbf{s} \cdot \mathbf{A} + f \frac{T_{\mathbf{max}}}{c} \hat{S} + 1$$

with switching function

$$\hat{S} = -\frac{c}{m} \frac{\boldsymbol{\lambda}_\mathbf{s}^\top \mathbf{B} \mathbf{B}^\top \boldsymbol{\lambda}_\mathbf{s}}{||\mathbf{B}^\top \cdot \boldsymbol{\lambda}_\mathbf{s}||} - \lambda_m \tag{12}$$

Therefore, throttling factor should be 1 (full on) for $\hat{S} < 0$, and 0 (off) for $\hat{S} > 0$. Moreover, it is possible to check from Eq. (6) that $\lambda_m \geq 0$ for the minimum-time problem with free final mass (because $\dot{\lambda}_m = -\partial_m \hat{\mathcal{H}} < 0$ and the free final mass condition implies $\lambda_m(t_f) = 0$), leading to $\hat{S} < 0$ for $t \in [t_i, t_f]$. As a consequence, f will always be 1 for minimum-time problems. Additionally, the extra degree of freedom in the minimum-time problem (t_f is unknown) requires the introduction of a transversality condition[12] setting $\hat{\mathcal{H}}\left(t_f\right) = 0$.

Gathering all the previous developments, the TPBVP derived from the first order optimality conditions for the minimum-energy/fuel problems can be stated as follows: to find the $\boldsymbol{\lambda}_i$ that satisfies the shooting function

$$\mathbf{Z}(\boldsymbol{\lambda}_i) = \begin{bmatrix} p\left(t_f\right) - p_f \\ e_x\left(t_f\right) - e_{xf} \\ e_y\left(t_f\right) - e_{yf} \\ h_x\left(t_f\right) - h_{xf} \\ h_y\left(t_f\right) - h_{yf} \\ \lambda_L\left(t_f\right) \\ \lambda_m\left(t_f\right) \end{bmatrix} = \mathbf{0}\,, \tag{13}$$

where the mappings $\mathbf{x}\left(t_f; \mathbf{x}_i, \boldsymbol{\lambda}_i, t_i\right)$ and $\boldsymbol{\lambda}\left(t_f; \mathbf{x}_i, \boldsymbol{\lambda}_i, t_i\right)$ come from the integration of Eqs. (2) and (6) with the controls $\mathbf{u}^* = [f^*\, \boldsymbol{\alpha}^*]$ derived from the PMP in Eqs. (8) and (11). The minimum-time problem presents the same structure, introducing an additional unknown, t_f, and associated algebraic condition, $\hat{\mathcal{H}}\left(t_f\right) = 0$, and setting $f^* = 1$ for all t.

2.4. Algorithmic implementation

The previous TPBVP is to be solved numerically using an adequate scheme. In this work a classic multiple shooting algorithm is employed, with a third-party root finder (Matlab's *fsolve* and its Trust-Region-Dogleg algorithm) and a custom Runge-Kutta Felhberg 7(8) propagator.

One key aspect regarding the propagator is the need to accurately locate the thrust switching events, that is, when the thruster should change its operation mode according to Eq. (11). To do this, we search for any change in the operation mode associated to the value of S at both ends of a successful integration step. If a change is detected it means a switching event takes place at an intermediate point inside the step, and the zero t_{switch} for the switching function is sought for using Newton's method. Because the convergence of Newton's method inside the interval cannot be guaranteed in general, a fall-back algorithm based on a combination of modified *regula falsi* (Anderson-Björk version) and bisection is also implemented. The initial guess for Newton's method is obtained using the values of S and its first derivative with respect to t at both endpoints of the step to construct a cubic Hermite interpolant, which is then solved analytically. Note that the information required to construct the interpolant is already provided by the RKF 7(8) except for the derivative of S at the end of the interval, so the added computational cost is small. The initial guess obtained from the interpolant is normally very close to the actual root, and the Newton's method takes just 2-3 iterations in most cases. It is important to highlight that each iteration requires taking a new RKF 7(8) step, involving 13 evaluation of the equations, so the use of an accurate initial guess to reduce the number of iterations has a noticeable impact on computational time for propagations with many switching events. Once the switching time has been determined, one RKF fixed step is taken up to t_{switch}, and the propagation continues for the new value of f^*.

The Trust-Region-Dogleg algorithm employed to solve the TPBVP requires the Jacobian of the shooting function (or a suitable approximation). Although it is possible to approximate it numerically using finite differences and BFGS updates, a more efficient and robust algorithm can be obtained by constructing it analytically from the State Transition Matrix (STM).[7] The STM maps small variations in the initial conditions $[\delta\mathbf{x}_i\ \delta\boldsymbol{\lambda}_i]$ to variations $[\delta\mathbf{x}\ \delta\boldsymbol{\lambda}]$ at a given time t, and its calculation requires deriving the variational equation and integrating it together with Eqs. (2) and (6). Because the STM is a 14x14 matrix, 196 additional differential equations have to be added, for a total of 210 equations. However, the straightforward derivation of the variational equations for this problem is not an easy task. Even though the procedure is conceptually simple, the resulting expressions turn out to be too cumbersome to handle even with the use of symbolic manipulators. This issue can be circumvented by taking advantage of the structure of the equations, obtaining general expressions for the variational equations in terms of $\mathbf{B}\mathbf{B}^\top$, $\mathbf{A}$ and their derivatives up to order 2.[15] The analytical derivatives for $\mathbf{B}\mathbf{B}^\top$ and $\mathbf{A}$ are then easy to determine with the help of a symbolic manipulator. One last aspect to consider regarding the STM is how to treat the discontinuities that appear in bang-bang control profiles. In this work, the strategy propossed by Russell[16] to obtain the composite STM for a trajectory with several bang-bang points is used.

Table 1. Environmental, thruster and spacecraft parameters.

Parameter	Value
μ [km^3/s^2]	398600.433
g_0 [m/s^2]	9.807
I_{sp} [s]	4000
m_i [kg]	675

Table 2. Departure orbital elements for all test cases.

Orb. Elems.	Departure
a [km]	29598.896
e [−]	0.000173
i [deg]	54.982
Ω [deg]	203.549
ω [deg]	272.857
θ [deg]	166.269

Table 3. Arrival orbital elements for each test case.

Orb. Elems.	Case 1	Case 2	Case 3
a [km]	31862.568	33006.338	33249.803
e [−]	0.071201	0.103189	0.109737
i [deg]	54.993	54.986	55.267
Ω [deg]	203.568	203.552	203.585
ω [deg]	256.033	39.376	30.013
θ [deg]	–	–	–
Δv_{imp} [m/s]	128.152	185.147	197.477
$t_{\text{re-entry}}$ [years]	98.403	76.003	68.408

Finally, the lack of information about the solution complicates the task of finding an initial guess for $\boldsymbol{\lambda}_i$ falling inside the convergence region of the solver. This can be addressed by solving the problem for a higher value of $T_{\max}$ first, and then performing a continuation in thrust until the desired nominal thrust is reached. The higher control authority increases the convergence region, allowing to find a valid initial guess by testing random values for $\boldsymbol{\lambda}_i$.

Now that a robust and efficient solver for the TPBVP has been defined, the minimum-fuel problem for given $T_{\max}$ and boundary conditions can be solved according to the following scheme

1. The minimum-time problem is solved performing a continuation in thrust, starting from a high enough value to ease the initial convergence of the TPBVP.
2. The minimum-energy problem is solved performing a continuation in thrust, for a final time taken longer than the time of flight for the minimum-time problem.
3. The minimum-fuel problem is solved performing a continuation in ε (starting from the minimum-energy solution), for the desired values of $T_{\max}$ and time of flight.

Under some circumstances, the continuation process may stall. In those cases, the freedom in the choice of θ_i is leveraged to restart it by updating the starting point within the departure orbit.

3. Test Cases

The proposed Galileo low thrust EoL disposal is now studied for three different test cases taken from the previous works by Armellin et al.[2] The departure orbital elements, common to all test cases, are compiled in Table 2, and the arrival orbital elements are shown in Table 3. The latter table also includes the

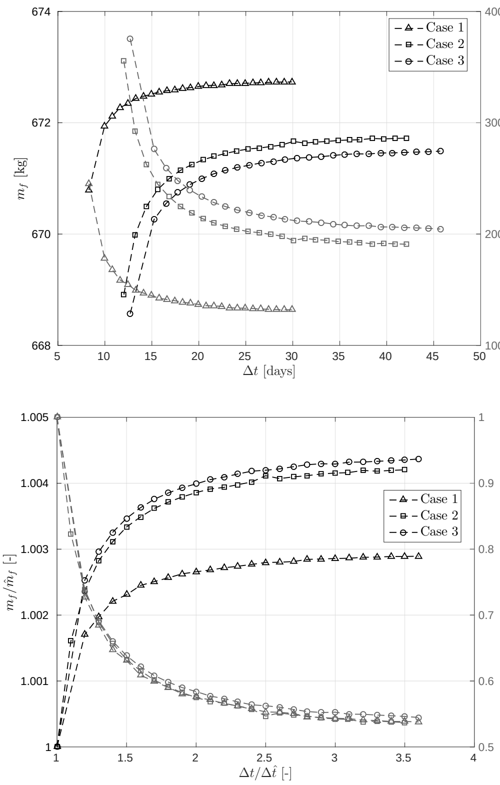

Fig. 1. Minimum-fuel m_f and Δv for the three test cases, for $T_{\max}$ = 230 mN and increasing times of flight Δt. Top figure corresponds to the physical values, whereas bottom figure is normalized with the minimum-time solution.

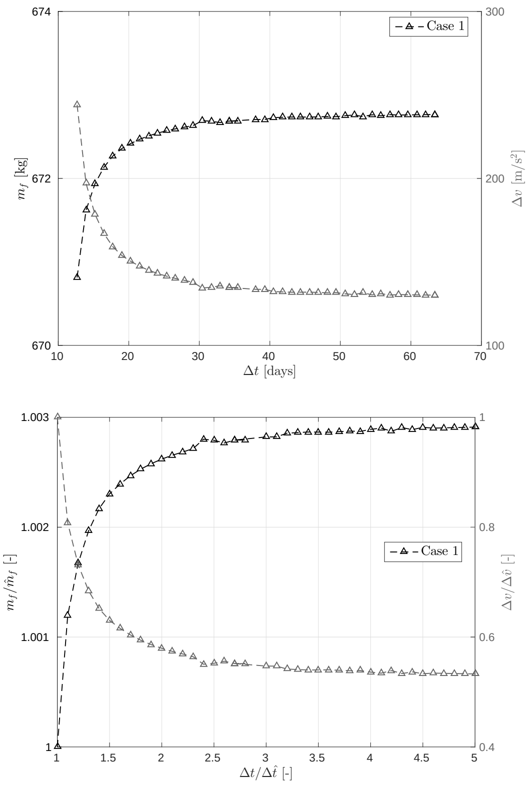

Fig. 2. Minimum-fuel m_f and Δv for test case 1, for $T_{\max}$ = 150 mN and increasing times of flight Δt. Top figure corresponds to the physical values, whereas bottom figure is normalized with the minimum-time solution.

re-entry time for each disposal orbit, as well as the Δv needed to reach it with an impulsive maneuver. Initial spacecraft mass is set to 675 kg, to be consistent with the dry mass of about 670 kg assumed by Armellin et al.[2] when obtaining the disposal orbits. Two different thrust levels are considered: 150 mN and 230 mN. The former is close to the 145 mN qualified by ESA for the BepiColombo,[5] while the latter corresponds to the maximum achievable thrust according to QnetiQ.[4] In both cases, a conservative value for I_{sp} of 4000 s is used. All the other relevant parameters are summarized in Table 1.

Figure 1 shows the minimum-fuel maneuver m_f and Δv for the three test cases at a maximum thrust level of 230 mN and increasing times of flight. As expected, the values for Δv are appreciably higher than for the impulsive maneuver (especially for times of flight close to the minimum-time solution), but the high I_{sp} of electric thrusters leads to small fuel consumptions. It is observed that fuel consumption, corresponding to $m_f - m_i$, is less than 5 kg in most cases for times of flight up to one and a half months, showing the practical feasibility of this EoL disposal approach. Additionally, increasing the time of flight with respect of the minimum-time maneuver up to a factor of 3.5 can provide reductions in the required Δv of around 45%.

Similar results are obtained for test case 1 at a maximum thrust of 150 mN, as shown in Fig. 2. The trajectory and throttling profile corresponding to a minimum-fuel transfer with t_f = 19.009 days (1.5 times the t_f for the minimum-time transfer) are represented in Figs. 3 and 4, respectively. It is straightforward to check that each revolution consists on one or two propelled arcs taking place around the centers of the orbit, with coasting arc in between. This is because the imposed orbital

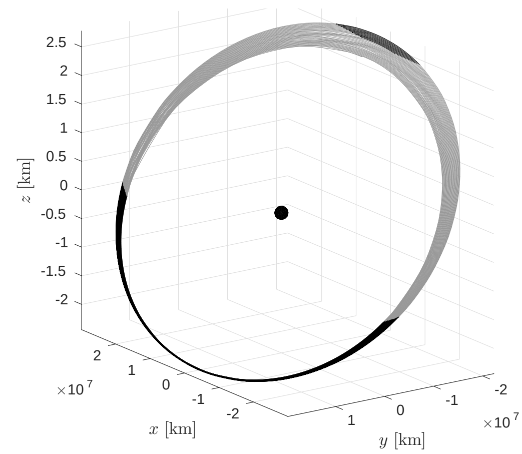

Fig. 3. Minimum-fuel trajectory for test case 1 with $T_{\max}$ = 150 mN and t_f = 19.009 days (1.5 times the t_f for the minimum-time transfer), in inertial reference frame. Coasting arcs are represented in gray, and propelled arcs in black.

maneuver changes the argument of perigee, not the orbit geometry,

4. Conclusion

The EoL disposal of satellites in the Galileo constellation using low thrust propulsion has been studied, leveraging previously computed orbits leading to the natural re-entry of the satellite due to lunisolar perturbation effects within a 100 years timespan. The low thrust transfers between the operational and disposal orbits have been designed using indirect optimization

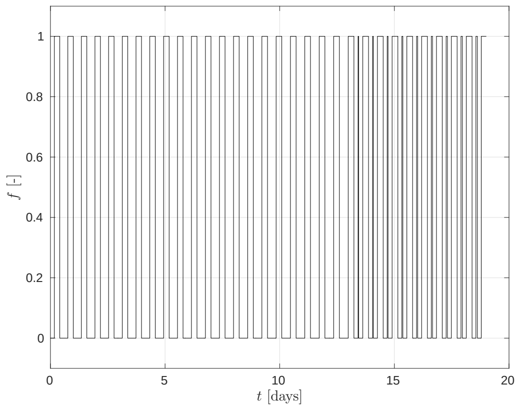

Fig. 4. Minimum-fuel throttling profile for test case 1 with $T_{\max}$ = 150 mN and t_f = 19.009 days (1.5 times the t_f for the minimum-time transfer).

methods and a MEE formulation for the dynamics. A special focus has been placed in applying different techniques to increase the robustness and efficiency of the indirect optimization algorithm, such as continuation, analytical derivation of the variational equations to propagate the STM, and precise and fast determination of thruster-switching events during the propagation. The use of MEE has allowed to express the orbit-to-orbit boundary conditions in a simple way, at the cost of more complex state and costate equations.

Numerical results for several representative test cases show fuel requirements of less than 5 kg for transfer maneuvers of up to one and a half months, proving the practical interest of this approach for the EoL disposal of future generations of Galileo satellites featuring electric propulsion. Furthermore, this strategy could be extended in future works to other constellations in the MEO region.

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