A Study of the Navigation for Spacecraft by Using the Modified Estimator

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The error budget analysis is presented which quantifies the effects of different error sources in the Earth-based orbit determination process when the orbit estimation filter is used to reduce radio metric data. The estimator strategy differs from more traditional filtering methods in the nearly all of the principal ground system calibration errors affecting the data are represented as filter parameters. The article reviews the fundamental concepts of reduced-order filtering theory, which are essential for sensitivity analysis and error budget development.

1. Introduction

Development of improved navigation techniques which utilize radiometric (Ranging and Doppler) data acquired from some ground stations have received considerable study in several years, as these data types are routinely collected in tracking, telemetry, and command operations. A sequential data filtering strategy currently under study is the orbit estimator, in which most if not all of the major systematic ground system calibration error sources are treated as estimated parameters, along with the spacecraft trajectory parameters. This strategy differs from current practice, in which the ground system calibration error sources are represented as unestimated bias parameters, accounted for only when computing the error covariance of the filter (estimator) parameters.

This article reviews the fundamental concepts of reduced-order filtering theory, which are essential for sensitivity analysis and error budget development. The theory is then applied to the development of an error budget for a Mars mission cruise scenario in which enhanced orbit estimation is used to reduce X-band Doppler and ranging data. The filter model is described and error budgets are given for two different strategies: X-band Doppler only, X-band Doppler plus ranging.

For this study, the filter model is assumed to be correct representation of the physical world.

2. Reduced-order filter

In some navigation applications, it is not practical to implement a full-order or the optimal filter when system model, with all major error and noise sources, is of high order. Use of reduced-order filter allows the analyst to obtain estimates of key parameters of interest, with reduced computational burden and with moderate complexity in the filter model. Thus, reduced-order or suboptimal filters are results of design trade-offs in which sources of error are most critical to over all system performance. Nevertheless, there are reasons for not always using a full-order optimal filter for spacecraft orbit estimation.

Some of reasons includes : (1) there may be a lack of adequate models for an actual physical effect; (2) certain parameters, such as the station location, may be held fixed in order to define reference frame and/or length scale; (3) if estimated, the computed uncertainty in model parameters would be reduced far below the level warranted by model accuracy.

2.1. Estimator evaluation

There are a number of error analysis methods which can be used to evaluate estimator (filter) models and predict filter performance. Reduced-order error analysis techniques enable an analyst to study the effects of using incorrect a priori statistics, data-noise/data-weight assumptions, or process noise model on the filter design.

If the filter is optimal, then the filter and truth models coincide. If the filter is suboptimal, then the filter model is of equal or lower order (i.e., reduced-order) than the truth model and possibly represents a subset of the states of the truth model. In practice, a fully detailed truth model may be difficult to develop and thus one typically evaluates a range of 'reasonable' truth models to assess whether the filter results are especially sensitive to a particular elements of the filtering strategy being used. The objective is to design a filter model to achieve the best possible accuracy, but which is also robust, so that its performance will not be adversely affected by the use of slightly incorrect filter parameters.

In a special case of reduced-order error analysis, various systematic error sources are treated as unmodeled parameters which are not estimated, but whose effects are accounted for in computing the error covariance of the estimated parameters. In a consider state analysis, the sensitivity of the estimated parameter set to various unmodeled consider parameters can be computed via partial derivatives of the state estimate with respect to the consider parameters set. The filter has no knowledge about the contribution the unmodeled parameters to the uncertainty in the state estimate since the modified covariance, which includes effects from both the estimated

and consider parameters, is not fed back to the filter.

2.2. Optimal and suboptimal estimator

Restricting the discussion to the filter measurement up-date equations, the mathematical model presented here is the estimator form of the measurement up-date.

Let $\hat{x}$ represent the state estimate and P represent the error covariance matrix. Using the convention that '(-)' denotes a pre-observation up-date value and '(+)' denotes a post-observation up-date value, the filter observation up-date equations for Extended Kalman type's estimator are given by

$$\text{State estimate} \quad : \quad \hat{x}_k^{(+)} = \hat{x}_k^{(-)} + \hat{K}_k \left[z_k - H_{xk}\hat{x}_k^{(-)} \right] \quad (1)$$

$$\text{Error covariance} : P_k^{(+)} = \left[I - \hat{K}_k H_{xk} \right] P_k^{(-)} \quad (2)$$

$$\text{Gain matrix} \quad : \quad \hat{K}_k = \alpha_k^{-1} P_k^{(-)} H_{xk}^T \quad (3)$$

where z_k is the observation vector defined by the measurement model, H_{xk} is observation matrix of measurement partial derivatives, I is simply the unit matrix, and $\alpha_k = H_{xk} P_k^{(-)} H_{xk}^T + W_k^{-1}$ is the innovation covariance. W_k represents the weighting matrix, the inverse of which is taken to be the diagonal observation covariance V_k; thus for $i = 1, \ldots, m$ observations, $W_k^{-1} \equiv V_k = diag \left[v_1, \cdots\cdots, v_m \right]$ for observation variances v_i. The filter equations described by Eqs (1) through (3) can be employed without loss of generality, since whitening procedures can be used to statistically decouple the measurements in the presence of correlated observation noise and obtain a diagonal V_k. The gain matrix K_k is used to up-date estimates of the filter parameters as each measurement is processed. And denote that Eq. (2) is valid only for the optimal gain $\hat{K}_k$

The use of Eq. (2) to compute the error covariance matrix has historically been suspect due to finit computer word length limitations. As a result, a utilized alternative is the stabilized form of the up-date, expressed as

$$P_k^{(+)} = (I - K_k H_{xk}) P_k^{(-)} (I - K_k H_{xk})^T + K_k W_k^{-1} K_k^T \quad (4)$$

Although this form of the covariance observation up-date is more stable numerically than Eq. (2), it requires a greater number of computations; however, a further advantage is that it is valid for the arbitary gain matrices; therefore, K_k in Eq. (4) need not be optimal.

In some cases, the observation up-date equation may also be deficient numerically. As a result, factorization methods have been developed to help alleviate the numerical deficiencies of the up-date algorithms. The details of the factorization procedures will not be discussed here; however, an important observation from the literature and critical to the general evaluation mode of the filter is the observation that Eq.(4) can be written in an equivalent form as

$$P_k^{(+)} = (I - K_k H_{xk}) P_k^{(-)} + \alpha_k (K_k - \hat{K}_k)(K_k - \hat{K}_k)^T \quad (5)$$

where K_k is an suboptimal gain matrix and $\hat{K}_k$ is the optimal gain matrix. This equation of the error covariance observation up-date is referred to as the suboptimal observation up-date since it includes a correlation based on the gain difference between the filter evaluation run and the original estimation run. In the general evaluation mode, the estimator uses suboptimal gains saved in an evaluation filter from an earlier filter which is run purposely with what is believed to be an incorrect model, in order to generate suboptimal gains. It is this strategy of the suboptimal observation up-date which will be critical to the error described in the following section. It is important to note that the time in the filter evaluation mode takes the same style as the original estimator time up-date, except that in the presence of process noise modeling parameters, the original estimator stochastic time constants and process noise (system noise) uncertainties are replaced with evaluation mode time constants and process noise terms.

3. Observation strategy and the estimator

3.1. Observation strategy

Observation data acquisition plan is assumed, containing several passes of two-way Doppler and ranging data per week. And also, the data schedule consisted of about 6 hours tracking pass of two-way Doppler and of about 2 hours tracking pass of two-way range from USUDA station basis from ME (Mars encounter) – 30 days to ME-10 days.

To account for observation noise, an assumed one-sigma random measurement uncertainty of 0.02 mm/sec was chosen for two-way Doppler, and for two-way ranging, the one-sigma random measurement uncertainty was assumed to be about 5 m. It should be noted that the data weights quoted here are for the round trip range-rate and range, respectively. Both data types were collected at a rate of one point every 10 min., and the noise variances were adjusted by an elevation-dependent function for USUDA station, to reduce the weight of the low elevation data; furthermore, no data were acquired at elevations of less than 13 deg.

3.2. The estimator

Table 1. summarizes the parameters which make up the filter model, along with a priori statistics, steady state uncertainties for the Gauss-Markov parameters, and noise densities for the random-walk parameters. All of the parameters were treated as filter (estimator) parameters and grouped into three categories: spacecraft epoch state, spacecraft nongravitational force model, and ground system error model. Effects of uncertainty in the ephemeris and mass of Mars were believed to be relatively small in this scenario.

Table 1. Estimation parameters (Assumed)

Estimate parameter	Uncertainty (one -sigma)
State vector	
Position element	1×10^7 km
Velocity element	$1 \, km/s$
Nongaravitational force	
SRP	10%
Anoumalous accelerations	1×10^{-12} km/sec^2
Range biase	5 m
Station location (U SUDA)	
Spin radious	0.05 m
Z-hight	0.05 m
Longitudu	3×10^{-9} deg

The simplified spacecraft nongravitational force model was used. There were filter parameters representing solar radiation pressure (SRP) forces as well as small anomalous forces due to gas leaks and attitude control thruster misalignments, and so on.

For processing the two-way range data, the filter model included a stochastic bias parameter associated with each ranging pass from the station, in order to approximate the slowly varying, nongeometric delays in ranging observations that are caused principally by station delay calibration errors and uncalibrated solar-plasma effects.

The station location covariance represents the uncertainty in the station location.

4. The error values

The purpose of developing an error budget is to determine the contribution of individual error sources, or groups of error sources, to the total navigational uncertainty. In general, an error budget is a catalog of the contributions of the error sources which contribute to errors in the filter estimate at a particular point in time, whether explicitly modeled in the filter or not. For the first analysis, it is assumed that filter is optimal, that estimator model is an accurate representation of the physical world.

In order to establish an error budget, it is necessary to compute a time history of the filter gain matrix for the complete filter and to subsequently use these gains in the sensitivity calculations (Eq. (4)) during repeated filter evaluation mode runs, in which only selected error sources or groups of error sources are 'turned on' in each particular run. In this way, the individual contributions of each error sources or group of error sources to the total statistics uncertainty obtained for all of the filter parameters for given radiometric data set can be established.

Using the reduced observation data schedule and the filter model derived for Mars mission scenario, orbit estimation error statistics were computed for Doppler-only and Doppler-plus-ranging observation data sets. The orbit estimation were propagated to the nominal time of Mars encounter and expressed as dispersions in a Mars centered aiming plane, or B-plane, coordinate system; specifically, the one-sigma magnitude uncertainty of the miss vector, resolved into respective miss components $\mathbf{B} \cdot \mathbf{T}$ (parallel to planetary equatorial plane) and $\mathbf{B} \cdot \mathbf{R}$ (normal to planetary equatorial plane. This plane definite Figure. 1.

In the B-plane ellipse, there are semimejor (SMAA) axis and semiminor (SMIA) axis. Where γ is the orientation angle of semimajor axis measured positive clockwise from $\mathbf{T}$ to $\mathbf{R}$.

Additional, the one-sigma uncertainty on the linearized time of flight (LTF). The LTF defines the time from encounter (point of closest approach) and specifies what the time of flight to encounter would be if the magnitude of the miss vector were zero. In the case, the errors were expressed as dispersion ellipses in the B-plane to graphically significant groups of error sources.
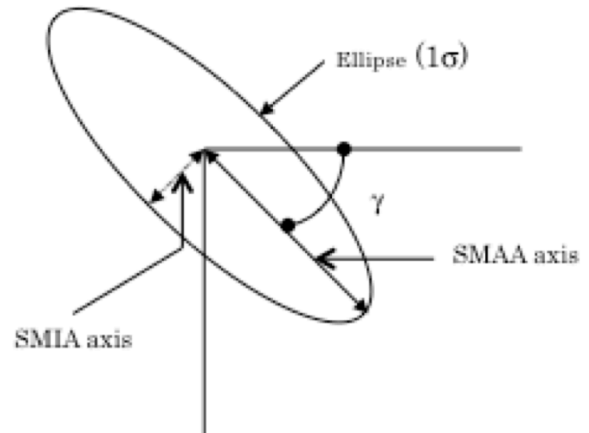

Fig. 1.   Definition of B-plane ellipse.

4.1.   In the case of 2-way Doppler only

With the reduced-filter, the 2-way Doppler data allowed determination of the $\mathbf{B} \cdot \mathbf{T}$ component of the miss vector to about 50 km and the $\mathbf{B} \cdot \mathbf{R}$ component of miss vector to about 25 km, with the LTF determined approximately 8 sec. These results summarized in Fig.2, which gives the magnitude of the B-plane error ellipse around the nominal aim point for the groups of the filter model error sources to the total statistical uncertainty, in a root-sum-square.
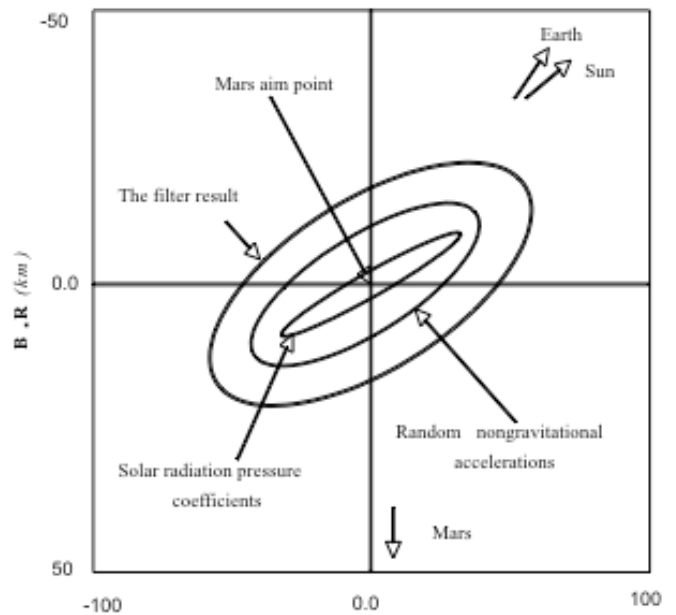

Fig. 2. The error ellipse on the B-plane for X-band 2-way Doppler only at closest approach

The most dominant error source groups were the random nongravitational acceleration, followed by solar radiation pressure coefficient uncertainty, and ground system calibration error. For this encounter phase, the direction of the Earth-spacecraft range is closely aligned with semmimajor (SMMA) axis   of the B-plane error ellipse. The Doppler data alone were able to determine this component of the solution to only about 55 km.

4.2. In the case of 2-way Doppler plus ranging

More one case in which both the 2-way Doppler plus ranging data were used, the $\mathbf{B} \cdot \mathbf{T}$ component of the miss vector was determined to about 6 km and the $\mathbf{B} \cdot \mathbf{R}$ component to about 5 km, with the LTF determined approximately 5 sec.

Similar to the results for the Doppler-only data strategies (See 3.1), random nongravitational accelerations were the dominant error source group.

In additional ranging data to Doppler data, the dispersion is reduced compare with only Doppler observation strategy. B-plane error ellipse are also provided (see Fig. 3.), illustrating the contributions of the major source groups to the total root-sum-square error and the orientation of the ellipses in the aiming plane. In this case, the accuracy with which the Earth-spacecraft range component at encounter was determined was roughly 12 km.
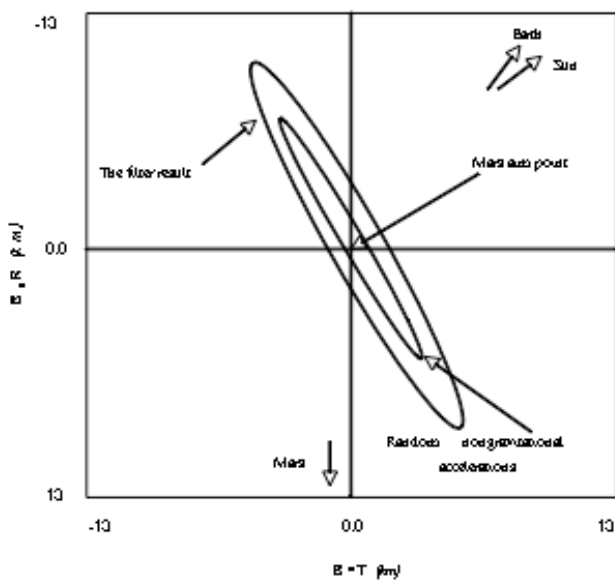

Fig. 3. The error ellipse on the B-plane for X-band 2-way Doppler plus ranging at closest approach

5. Sensitivity analysis

The results of the linearity assumptions used to develop error budgets is that sensitivity values can readily bee generated. These values graphically illustrate the effects of using different prescribed values of the error source statistics on the estimation errors, with the assumption that the reduced-order filter.

The procedure for sensitivity values development is repeated here for completeness:

(1) Subtract the contribution of the error source under consideration from total mean-square navigation error.

(2) To compute the effect of changing the error source by a preset scale factor, multiply its contributions to the mean-square errors by the square of the scale factor value.

(3) Replace the original contribution to mean-square error by the one computed in the previous step.

(4) Take the square root of the newly computed mean-square error to obtain the total root-sum square navigation error.

Several cases were used to generate sensitivity curves for the major groups of error sources in the filter (estimator). Fig. 4. and Fig. 5. give the sensitivity curves for the random nongarvitational accelerations and illustrate the sensitivity of this error source group to various scale factor values. Random nongravitationnal acceleration dominated the error budget in two data strategy cases considered.
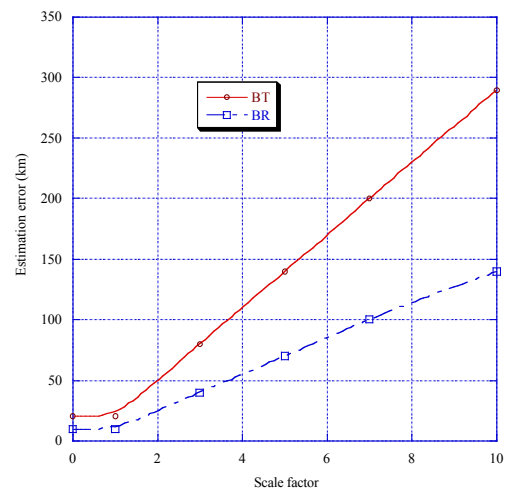

Fig. 4 Sensitivity of he estimation error to perturbation

of random nongravitational accelerations

(X-band two-way Doppler only)

As seen from the figures, a quadratic growth in the sensitivity is evident for scale factor values ranging from 1 to 3, and a nearly linear growth is exhibited for scale factor values ranging from 4 to 10. On average, for two data strategies considered, an order of magnitude increase in the preset scale factor resulted in about a factor of three to six increase in the root-mean-square estimation errors.
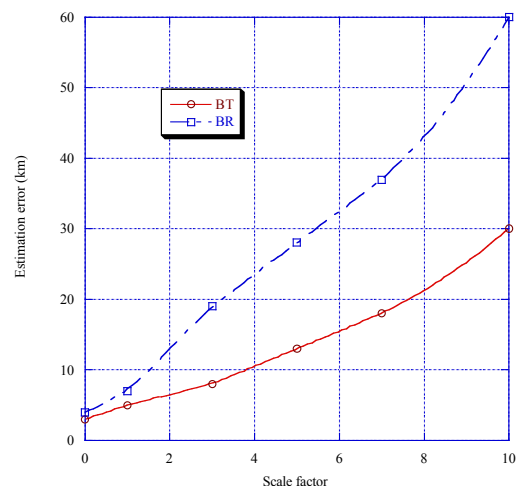
Fig. 5 Sensitivity of he estimation error to perturbation of random nongravitational accelerations (X-band two-way Doppler plus ranging

5. Conclusion

A sensitivity analysis was conducted for a reduced-order filter referred to as the enhanced orbit estimation filter. In practice, the enhanced filter attempts to represent all or nearly all of the principal ground system error sources affecting radiometric data types as filter parameters. A reduced-order filter technique were reviewed and utilized to perform the sensitivity analysis, and, in particular, to develop navigation error budgets for two different data acquisition strategies.

Error budget performed for the assumed mission strategy revealed that the most significant error source for two data-acquisition strategies studied was spacecraft random nongravitational accelerations, indicating that, for the reference error model, the enhanced filter is most sensitive to mismodelling of small anomalous forces affecting spacecraft. These results suggest that if high-precision navigation performance is to be achieved, the error sources requiring the most accurate modelling are spacecraft nongravitational accelerations error.

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