Initial Results of a New Method for Optimizing Low-Thrust Gravity-Assist Missions

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Further exploring our solar system is continuously becoming more challenging, thus gravity-assists and low-thrust propulsion are typical mission enablers. Combining both has the potential to improve performance even more. Typically, optimization for low-thrust gravity-assist missions is conducted only for the trajectory. To improve results, it is however desirable to include finding the gravity-assist in the optimization. This paper presents a method, able to evaluate a large number of mission candidates, defined by gravity-assist maneuvers and the trajectories in between, with the gravity-assist partner as one control variable. A heuristic search is conducted under application of a shape based trajectory model. The paper explains the structure of the control variables and their repercussions on the optimization and presents the initial results obtained for three major questions: Is the method overall usable? Can the search space be pruned with constraints based on the maximum obtainable Δv and the pool of possible gravity-assist partners? Can evolutionary algorithms be used to optimize such missions with the given method? The basic method has been effective in finding optimal gravity-assist partners for single-gravity-assist missions and shows potential for multi-gravity-assist missions as well. Derived from the optimization performance some conclusions about the search space topography are drawn.

Key Words: Gravity-assist Sequencing, Low-thrust Trajectories, Optimization

Nomenclature

a	:	semi-major axis (of an elliptical orbit), resp. coefficient of trajectory model
b	:	coefficient of trajectory model
c	:	coefficient of trajectory model
C_j	:	Jacobi integral
$C_j{*}$	:	modified Jacobi integral
d	:	coefficient of trajectory model
δ	:	turning angle/ deflection of velocity
e	:	eccentricity, resp. coefficient of trajectory model
f	:	coefficient of trajectory model
F	:	number for weighting of mutation
g	:	coefficient of trajectory model, number of generation
Δv	:	velocity change of a mission
i	:	inclination
μ	:	gravitational parameter
n	:	population number
R	:	orbital distance from bary-center
$\vec{T}$	:	thrust acceleration (vector)
θ	:	trajectory angle, describing progress
$\vec{V}$	:	velocity (vector)
$\vec{x}$	:	solution candidate (vector)

Subscripts

0	:	initial property
i	:	population number
pl	:	planet's property
r1,…	:	random population number

1. Introduction

Ever since the spectacular *Voyager* and *Pioneer* missions, gravity assists[1)] are a typical method to improve mission performance or are even mission enablers. More recent examples are *Rosetta*, *Hayabusa* and *Dawn*, missions targeting small bodies in various regions of the solar system.

Due to their excellent efficiency, made evident by specific impulses above 1000 s[2)], low-thrust propulsion likewise can act as mission enabler and in general improves mission performance.

Combining both has the potential of improving mission performance even further, thus allowing missions with large Δv demands to be conducted with a manageable amount of propellant, just like *Dawn* and *Hayabusa*.

The nature of low-thrust trajectories, however, makes their optimization more challenging than for impulsive thrusters. For impulsive thrusters, any maneuver is defined by three dimensions: two thrust angles and the thrust magnitude.[3)] Following from the respective energy state after a maneuver, the subsequent trajectory is defined and set (not considering perturbation forces at this point). For a low-thrust mission, for each point in time, these three dimensions exist likewise. Because time is continuous and infinite, i.e. an infinite amount of points in time describe any trajectory or part of it, the solution space of the control variables associated with the low-thrust trajectory is also infinite.[4)]

As for many optimization problems, the search space topography is unknown and therefore typically heuristic methods are used for optimizing low-thrust trajectories.[3)]

For optimization of low-thrust gravity-assist trajectories, the sequence of gravity-assist partners (i.e. planets or moons) is typically fed into the optimizer and not part of the optimization process.[5,6)] Therefore, it requires a new method of incorporating such gravity-assist partners as part of the optimization to facilitate a complete and thorough examination of the search space and not restrict the search on certain areas. Such a method further allows analysis and review of the complete search space topography.

This paper proposes a new method potentially capable of examining the whole search space. As a first step to find a suitable way of optimizing and planning low-thrust gravity-assist sequences, methods of sequencing impulsive-mission gravity assists have been reviewed, which led to the application of Tisserand's Criterion for mapping sequences. In this paper it is shown that this energy relation cannot be used for missions under continuous (low) thrust and an alternative method is proposed. The steps to derive this method and preliminary results have been published in Ref. 7) to 9). This paper presents an update on the results and method. For further discussion, the formulations assume that the gravity-assist perspective is regarding Sun, planet and a spacecraft. In a more general perspective these terms are interchangeable with central body, gravity-assist partner and gravity-assist body. For clarity however the previous mentioned perspective is used in formulations, omitting other possible scenarios, which is not meant to represent lack of general validity of the statements.

2. Tisserand's Criterion Use for Low-Thrust Missions

A prominent method for mapping out gravity-assist sequences of impulsive missions are so called Tisserand graphs. These graphs link a heliocentric orbit's energy (resp. orbital period, which are both just depending on the orbit's semi-major axis) and pericenter distance with the planetcentric orbital energy expressed by the hyperbolic excess velocity during a flyby. They have been developed by two independent research groups,[10,11] and are based on the energy relation called *Tisserand's Criterion*.

This relation has been formulated by Felix Tisserand in the 19th century, to identify comets he observed before and after a close encounter with Jupiter.[12] It is a form of the Jacobi integral and is written as:

$$C_J = \frac{R_{pl}}{a} + 2\sqrt{\frac{a(1-e^2)}{R_{pl}}}\cos i = const, \qquad (1)$$

where R_{pl} is the solar distance of the planet, a the semi-major axis of the spacecraft and e and i its eccentricity and inclination respectively. It should be noted that in many formulations of Tisserand's Criterion, the semi-major axis a is scaled to the solar distance of the planet, which then does not appear visibly.

This equation describes the energy of the relative motion and is based on the Jacobi integral of the motion in the *restricted, circular three-body system*.[13] Since it is constant, it sets a constraint on the orbit around the Sun after a gravity assist and thus the possible orbit is not arbitrarily selectable for a mission design, but has to fulfill Eq.(1). It is a state quantity and thus can be evaluated a priori.

The derivation of the criterion within the restricted, circular three-body system places it in the environment of an ideal case. The assumptions of the restricted, circular three-body system are that the involved bodies have constant masses and circular orbits. For the derivation of Tisserand's Criterion it is also assumed that the distance to the planet is large and therefore its gravitational influence – once past the gravity assist – is small in comparison to the Sun's influence. And most importantly it is assumed that only gravitation is acting in this system. These assumptions are not congruent with the real solar system, especially not in the context of a thrusting spacecraft. The premise of only gravitation acting in the system is especially violated by missions applying low-thrust propulsion continuously.

While an impulsive maneuver places a spacecraft on a fixed trajectory once the maneuver is took place, continuous thrust is associated with a continuous change of orbital energy and thus the trajectory.

The deviation of the actual solar system and application of thrust from the ideal case have been analyzed in previous papers. Besides the effect of thrust, the strongest effect can be attributed to non-circular orbits of the planets around the Sun, mostly for Jupiter. Deviations in the numerical value for Tisserand's Criterion of up to 25% have numerically been determined by a search with random orbital parameters for a sample spacecraft.[7] Allover however, the energy input by the continuous thrust is the strongest deviation as it can easily reach about 80% of the total energy for typical missions (e.g. from Earth to Jupiter).[8] Consequently the effect of the thrust cannot be disregarded as the application of Tisserand's Criterion would suggest, even though natural errors of up to 25% have been found.

Therefore, the application of Tisserand's Criterion requires to regard the low-thrust effect in the criterion itself. With similar steps as for the original criterion, a correction term can be formulated, leading to a modified Jacobi integral for the motion under thrust (note this formulation applies for any magnitude of thrust and is not restricted to low-thrust propulsion):[8]

$$C_j^* = C_j + 2\int \vec{V}\cdot\vec{T}\,dt. \qquad (2)$$

The Jacobi Integral C_j is modified by an integral of the vector product of velocity $\vec{V}$ and thrust acceleration $\vec{T}$. While further simplifications are possible, Eq. (2) contains in any case a modified term, which is no state quantity. To evaluate it, the actually trajectory, especially the thrust and velocity histories need to be known. The thrust can be assumed to be e.g. tangential or parallel to the velocity vector and constant, but the velocity vector history is depending on the actual trajectory. Assumptions about the velocity and also thrusting time interval (dt) would already define the trajectory, which then would not be optimal (only by accident).

Over the course of a trajectory, Eq. (2) continuously changes its value due to the correction term. Consequently it is no longer a clear constraint on possible orbits. A priori the possible gravity assists cannot be mapped out.[8]

Tisserand's Criterion is therefore not applicable for low-thrust mission planning in a similar manner as for impulsive missions. While it can still be regarded as true for the actual gravity assist, any long duration – resulting in a non-negligible value for Eq. (2) – of the thrusting (in comparison to the overall mission time) is a unneglectable violation of the criterion's premises.

Therefore an elegant planning of gravity assists analogously to the impulsive maneuver case, is not possible. The search for the sequence has to occur heuristically instead.

3. Optimization Variables and Method

As explained in Section 2, the methods used for impulsive mission sequencing cannot be used, an alternative is necessary. To define a new method, the respective variables associated with gravity assists are reviewed. The optimization variables for a gravity-assist mission include those of the trajectory model, controlling thrust and thus the outcome of the trajectory. In addition, the gravity assist(s) add further variables for definition of the actual maneuver, e.g. the deflection (turning angle) δ.

For the purpose of this paper, a planar shape-based model developed by Wall and Conway[14] has been applied to define the trajectory. This model uses a polynomial to approximate a low-thrust trajectory in the form of:[14]

$$R(\Theta) = \frac{1}{a + b\Theta + c\Theta^2 + d\Theta^3 + e\Theta^4 + f\Theta^5 + g\Theta^6} \qquad (3)$$

where θ is the angle describing the progress on the trajectory (similar but not identical to the true anomaly, as θ is 0 at the start of the trajectory and can continue up to a value depending on the target bodies position at arrival) and a to g are coefficients used to adapt the polynomial to constraints, e.g. the departures state (described by the velocity and position vectors). The simplicity of the model allows a fast evaluation of a large number of trajectories in a short amount of time. It is an approximation, circumventing the need for propagation and integration.

For the purpose of this work, each mission has been divided into segments and each segment described by a polynomial as given in Eq. (3). The segments are defined by an initial state and a final state, containing the respective body, hyperbolic excess velocity and deflection for a gravity-assist encounter. Several segments, depending on the amount of gravity-assist partners, then describe a whole mission. The first segment of the mission contains the starting body (providing the position) and velocity and the last segment's final body is the mission's target body. Depending on whether or not it is a flyby or a rendezvous the hyperbolic excess velocity has a value unequal to or equal to zero.

3.1. Control Variables of the Problem

The described trajectory model uses the variables Number or Revolutions (around the Sun), N_{rev}, Time of Flight, *ToF*, and Launch Date, *LD*, to control the trajectory (i.e. they defined the coefficients a to g[14]). N_{rev} basically sets a constraint on thrust. The coefficients in Eq. (3) are calculated based on mission constraints, e.g. arrival date, and match the trajectory to its mission purpose, e.g. the respective arrival body at a specific time (defined by launch date plus flight time).

Each mission is defined by the variables Mission Time of Flight, *MToF*, Mission Launch Date, *MLD*, and Mission Number of Revolutions, MN_{rev}. As the bodies are the "handover points" between segments, a segment's final body is the next segment's initial body. Its arrival date is the next segment's launch date and so on. Also the segments' variables are not independent of the mission variables. The sum of all segment's *ToF* values has to add up to the *MToF*, for example.

The variable describing the respective gravity-assist partner, is simply an integer number, where Mercury is represented by the value 1, Venus by 2, Earth by 3 and so on.

However, the variables describing the segment's gravity assist are again not independent of the mission as such. A flight time fitting for a transfer to Mars would be not very suitable to reach Jupiter, for example. These variables are sensitive to the variable of the gravity-assist partner and this variable is discreet. Thus it has a strong influence on the variables describing the gravity assist.

These are the hyperbolic excess velocity (in all coordinates, which are x and y here) and the turning angle, transforming the velocity vector from its incoming direction to its outgoing direction (which then modifies the heliocentric velocity vector).

Therefore, two kinds of variables are defined: global and local. The former are freely selected and can be used for evolutionary search methods, whereas the latter cannot be freely selected due to their interdependency with other variables and describe the gravity-assist situation. They are reinitiated for each step in the search.

3.2. Search Method

With the variable structure as presented above, the following steps for the optimization are applied:[9]

1. Set constraints for mission (e.g. starting body),
2. optimize a no-gravity-assist trajectory (one segment) as benchmark for the search,
3. create a population of solution candidates for each value of the number of gravity-assists from 1 to a (user specified) maximum number, with random initial values for all variables, where:
    a. constraints and variable interrelations are observed to create a valid mission sequence,
    b. gravity-assist effect is considered by application of the turning angle on the planetcentric approach/ departure velocity (note this is the assumption of energy conservation),
4. optimize the solution candidates within each population, recombination of mission global variables (see 3.1) occurs, mission local variables are set in dependence to global and local variables,
5. compare best solutions of all populations (incl. the no-gravity-assist benchmark) to find final solution.

The usage of populations defined by the number of mission segments allows exchange of variable values between the solutions within a population as applicable for evolutionary algorithms and the benchmarking ensures that a selected gravity-assist trajectory is not worse performing than a non-gravity-assist trajectory.

4. Calculation Examples

To investigate the usefulness of the method, calculations have been conducted, investigating different algorithms and settings. For the calculations shown here, the example mission has been a transfer from Earth to Jupiter.

More details can be found in Ref. 9).

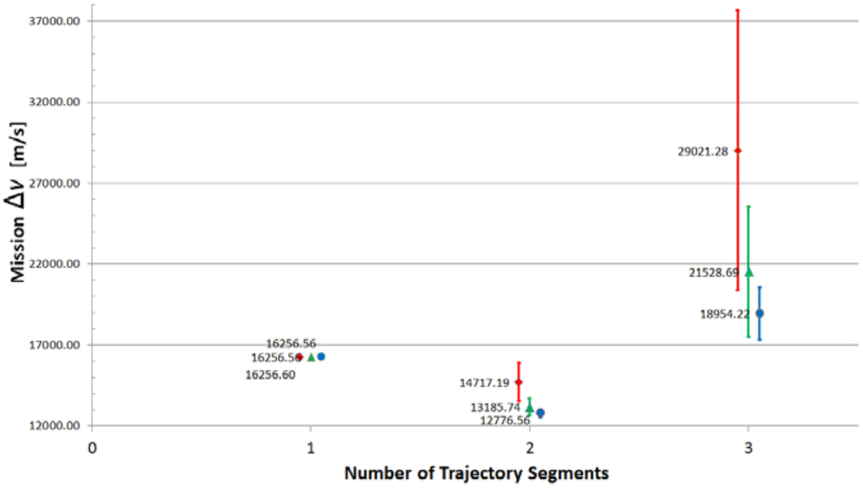

Fig. 1: The average results for calculations with Differential Evolution and their standard deviation. Resulting mission Δv over number of segments for 50 (red, diamond), 200 (green, triangle) and 500 (circle) population members.

4.1. Investigating Differential Evolution

One algorithm that has been tested, was Differential Evolution, due to its robustness.[15)] The major investigated question was if an evolutionary algorithm can be applied to the method in Section 3.2, despite the fact that not all variables can be used evolutionary (see Section 3.1). Also, the influence of the population size on the solution quality and if the results have been reliable, i.e. can be found repeatedly, has been analyzed.

Differential Evolution creates a mutation vector out of three existing solutions according to the pattern:

$$\vec{x}_i^{g+1} = \vec{x}_{r1}^g + F \cdot (\vec{x}_{r2}^g - \vec{x}_{r3}^g), \quad (4)$$

here g represents the number of the generation, i the index of the vector (from 1 to n, with n being the number of solution candidates), $r1$ to $r3$ random indexes and F is a real, constant factor ϵ [0,2], called differential weight. Variations of this definition include a larger number of vectors used for creating the difference vector.

Out of this mutation vector, a trial vector is created, where for each vector entry it is randomly determined (via the so called crossover constant) whether or not the mutated variable is accepted. At least for one entry this occurs automatically, ensuring that always a new solution is created out of an existing one. Details can be found in Ref 15).

4.1.1. Settings

The stopping criterion has been 1000 generations resp. four generations without replacement in the whole population. The launch date window has been 360 days, the flight time interval 1000 days, the crossover constant 0.75, the differential weight 1 and the maximum number of gravity assists has been 2. Population size has been varied between 50, 200 and 500, with 10 calculations each.

4.1.2. Results

The average results for all population sizes are shown in Fig. 1. It can be seen that the no-gravity-assist solutions (1 segment) do not change for different population sizes, whereas for gravity-assist missions the solution quality increases, i.e. the Δv decreases. Overall the best solution obtained has been 12.1 km/s for a single-gravity-assist trajectory (2 segments), for a gravity-assist at Earth. This solution is given in Fig. 2. It can be seen that initially the spacecraft remains close to Earth’s heliocentric orbit, before using a flyby at Earth to leave on an outwards course towards Jupiter.

For a population size of 200 and 500 the single gravity-assist missions always produced the same gravity-assist encounter: Earth at an encounter date of 56481 MJD (21.84 standard deviation), including the best solution.

Multi-gravity-assist missions produced no such unique encounter, however the majority of missions favored Earth as first gravity assist partner and Mars as second.

The number of generations never reached the limit of 1000. For no-gravity-assist missions, the generation number has been on average about 130 for all three population sizes, for single-gravity-assist missions it has been 86, 251.3 and 582 for the population sizes of 50, 200 and 500. For multi-gravity-assists they have been 93.6, 342.2 and 580.

4.2. Calculations Investigating Constraints

To further improve obtained results, it was also investigated which constraints on the search space can be useful. A simple analysis can reveal that the maximum obtainable Δv_{max} for a gravity-assist is:[8)]

$$\Delta v_{max} = \sqrt{\frac{\mu_{pl}}{R_{per}}}. \quad (5)$$

It depends on the gravitational parameter μ_{pl} of the respective planet and the pericenter distance R_{per} of the respective flyby trajectory. The hyperbolic excess velocity for which this maximum occurs is identical to Δv_{max}:

$$v_{\infty,max\Delta v} = \sqrt{\frac{\mu_{pl}}{R_{per}}}. \tag{6}$$

This means, once the flyby pericenter distance is determined, the maximum obtainable Δv_{max} is immediately clear.

Therefore, it is possible to restrict the search into regions around this maximum, to ensure a large gain from each gravity assist and thus reduce the number of gravity assists (and thus likely flight time). However there is no guarantee that a locally "optimized" gravity-assist outcome leads to an overall best sequence. Therefore it has been tested, whether or not this constraint improves solution quality.

Likewise it has been tested if it is beneficial to the solution quality, to restrict the selection of gravity-assist partners to the vicinity of the current segment's planet and prevent extensive "jumping" from one planet to the other. The variable of the gravity-assist partner identification is thus limited to a certain change, e.g. an increase or decrease maximum of one.

4.2.1. Settings

Calculations have been conducted with Differential Evolution, a population size of 100 and similar mission settings as before (see Section 4.1.2), including 1 to 2 gravity-assists. The constraints using the "region" around Δv_{max} has been varied in 10% steps from 0 to 100%, the constraint regarding the partner pool has been varied between 1 and 2 in both directions, i.e. inwards and outwards. In a further set of calculations the respective best results have been combined, i.e. a region of 20% and a constraint of 1 step in- and outwards.

4.2.2. Results

The calculation results have been between 14.2 and 15.9 km/s on average for one-gravity-assist missions, whereas the standard deviation was reduced by 65% (down to 348 m/s) compared to not applying the constraint when using a region of 20%, whereas the mission Δv was lowered by 190 m/s. Using a region of 40% led to a standard deviation of 448 m/s and a mission Δv improvement of on average 360 m/s vs. the calculations not using the constraint.

Applying a constraint regarding the planet partner pool did not show a clear behavior. The best result when only using the constraint regarding partner pool, occurred for a restriction of 2 in both directions, leading to an average Δv of 14.2 km/s with a standard deviation of 1.22 km/s.

5. Discussion

Regarding single-gravity-assists the usage of the proposed method showed significant improvement of the mission Δv, in the best case of up to 20%. Similar improvement has not been shown for multi-gravity-assists.

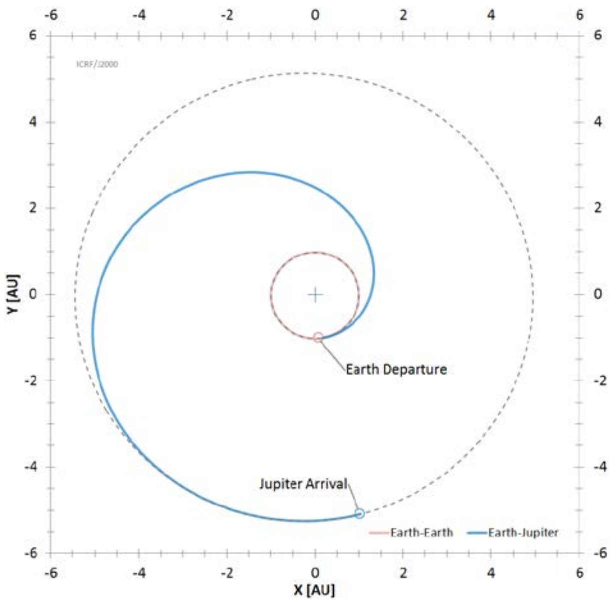

Fig. 2: The best obtained trajectory for an Earth-Jupiter mission (12.1 km/s mission Δv), with a single gravity assist at Earth. The first segment is shown in red, the second in blue, ending at Jupiter. The planetary orbits are depicted in dotted lines for reference.

5.1.1. Performance of Evolutionary Algorithm

In general, there has been a significant improvement of the mission Δv by using the global variables of MTOF, MN_{rev}, MLD and the gravity-assist partner ID for evolutionary optimization. It has been shown that an increase of population size also leads to an improvement of the solution quality, which is linked to the increase of diversity. A larger population increases the amount of variable information that can be used for evolving the solutions and therefore solutions improve. Also the standard deviation is reduced, i.e. the reliability of finding similar results. Therefore, the convergence to the global optimum is ensured, instead of a local optimum. This interpretation is further supported by the increase in used generations for increasing population size.

The increase of the population size, leading to an increase of available variable information, means that there is a larger chance to find solution improvement and thus replacement. Therefore the optimization runs for more generations than before.

The success regarding solution improvement on one-gravity-assist solutions over no-gravity-assist solutions, shows that the global variables are suited for evolutionary search and thus have a certain dominance in the problem. However, their influence expectedly decreases with an increasing number of gravity-assists, because of the real valued variables, associated with the gravity-assists, added to the problem, e.g. the hyperbolic excess velocity.

It becomes more difficult for the algorithm to find useful and beneficial information. A further obstacle is that only the global variables are used for evolution. Local variables are reinitiated for each iteration. This means that there is likelihood that possibly beneficial global variable values are not retained, unless combined with a good variable set for the

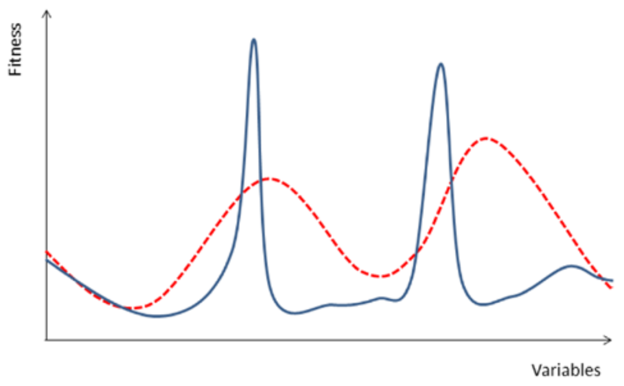

Fig. 3: A sketch of a 2D projection of search space topography regarding fitness over possible variable values, showing differences in steepness based on small numbers of gravity-assists (red/ dashed line) and large numbers of gravity-assists (blue/ solid line).

local variables. The chance for such a good combination increases with the population size and therefore an increasing population size leads to better solutions.

But also the search for multi-gravity-assist mission shows solution improvement to the same order of magnitude as the no-gravity-assist solution, but cannot improve it.

5.1.2. Performance of Constraints

The usage of the described constraints is not always leading to better solutions, but it reduces the standard deviation. This means the effect is not finding better solutions, but more similar solutions. This is expected – finding similar solutions results from constraining the search space to certain areas. This removes very bad results from the evaluation, but also potentially very good results. The effect is similar, but not the same, as for an increase of population size. Such an increase also means an increase in computation time, therefore using the constraints can lead to time savings. On the other hand, the constraints have to be carefully evaluated and tested out before usage, to ensure that it does not prevent convergence to the optimal solution.

5.1.3. Search Space Topography

Especially the decrease of solution quality with increasing numbers of gravity-assists, gives a hint on the search space topography. The increase means the probability to find good variable combinations drops, as the number of variables increases as well. However the possible fitness improvement, due to more Δv gain by several gravity assists, means that the peak of the optimum is also larger, yet the width is narrower, due to the reduced probability of finding beneficial variable combinations. This is illustrated in Fig. 3.

6. Outlook

The presented method has only been applied planary and was successful mostly for single gravity assists. Consequently the reliability of the optimization needs to be improved, i.e. its convergence.

Similarly, the trajectory model needs to be enhanced to involve all three dimension to allow more meaningful missions, e.g. also covering polar orbits or small body missions.

7. Conclusion

The results of an optimization method for low-thrust gravity-assist sequences have been shown. They have produced a Δv improvement of more than 20% for single-gravity-assist missions, but could not exceed no-gravity-assist solutions for multi-gravity-assist missions. Therefore, method improvement is necessary as well as an enhancement to three dimensions.

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