Satellite Constellations for Altimetry

By Alain Lamy[1)] and Josiane Costeraste[1)]

[1)] *CNES - Centre National d'Etudes Spatiales, Toulouse, France*

The framework of the analysis presented in this paper is the improvement of altimetry services in the 2025-2030 time frame by a better time and space coverage of the Earth (or rather ocean) surface. The typical expected revisit time is 5 days for ocean structures of size typically equal to 50km. Two types of instruments are considered: nadir altimeter or wide swath interferometer. The performance index used is based on a longitude-time coverage map. A criterion of 80% coverage with a probability of 0.8 has been chosen. The paper details the criterion and methods used. Various results and sensitivity analyses are also presented.

Keywords: Satellite constellations, Coverage, Revisit time, Altimetry

Nomenclature

N, P, Q	Repeat orbit parameters: (N, P, Q: integers, P < Q) Repeat Period = Q (Earth revolutions) N * Q + P orbit periods per repeat cycle.
D	Size (diameter) of ocean structures
F	Swath width
SSO	Sun-Synchronous Orbit

1. Introduction

The objective of the study is to identify candidate satellite constellations dedicated to mesoscale ocean circulation in the 2025-2030 time frame.

Only geometric aspects are considered in this study (instrument/satellite error budget is supposed in agreement with the capacity to detect the targeted ocean structures). Two types of instruments can be accommodated: nadir altimeter and wide swath interferometer.

The objective is first to define a performance index for space-time coverage. Then various satellite constellations will be analyzed. Each configuration is defined by the number of satellites, the number of planes, the instrument type (nadir or wide swath) or the satellite distribution in the different planes depending on inclination and altitude. A sensitivity analysis is also conducted in order to better understand the major factors the performance depends on.

2. Requirements and hypotheses

Based on the analysis of the required operational tools and models in the targeted time frame, the objective has been defined as follows: observe ocean structure of size 50 km with a revisit time of 5 days.

The orbits are supposed circular with an altitude between 500 and 800 km. They may be Sun-synchronous or not. Note that there is some debate about whether or not Sun-synchronous orbits are adapted to altimetry but that's not the point here.

3. Coverage criterion.

3.1. Detection/observation criterion

<u>Nadir</u>**:**

Ocean structures are idealized in the study: they are circular, and have a known diameter (D). Moreover the structures are not moving on Earth surface.

One considers the ascending and descending passes at a given latitude. An ocean structure is supposed to be detected if the distance to the center during the pass is smaller than some threshold = f * D/2 where f is between 0 and 1. This is equivalent to saying that the portion of Earth surface covered at that latitude is the same as that of a pseudo swath of width f * D. This is illustrated in figure 1.

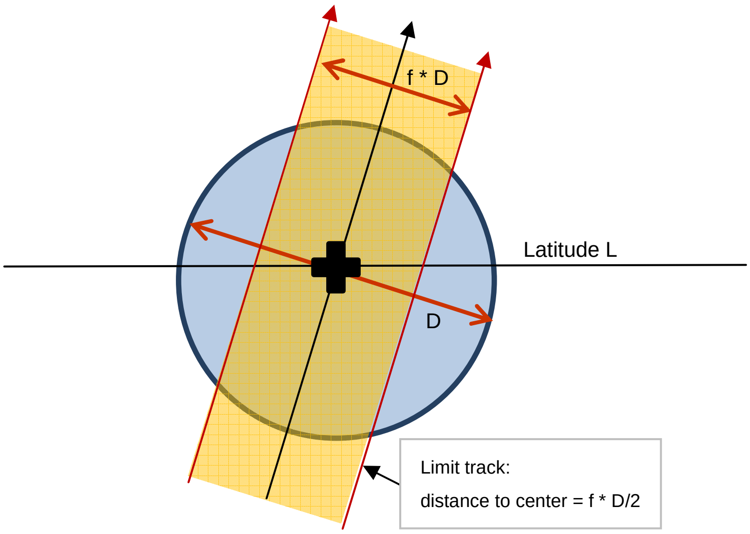


Figure 1: Detection of ocean structure

The value f = 0.5 has been proposed. This value was first estimated considering in-flight altimeters (JASON satellites in particular).

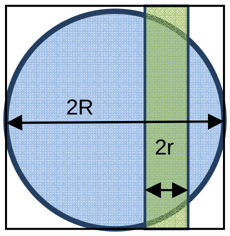
A similar value for f can be determined using a theoretical approach:

- Exponential weighting of the ocean structure: 1 at the center, 0.1 at the edge (1 radius from the center)
- Exponential weighting of the actual instrument swath: 1 at the center and 0.1 at the edge

The criterion to be met is then expressed by the "C/S" ratio, where C is the covered area (area of the structure that is seen), and S the swept-through area. More precisely:

$$C = \int_{X-r}^{X+r} \left(\int_{-R}^{R} e^{-\frac{\lambda^2 (x^2+y^2)}{R^2}} dy \right) dx$$

$$S = 2 * R * \int_{X-r}^{X+r} e^{-\frac{\lambda^2 x^2}{r^2}} dx$$



The 2 integrals are computed on the instrument swath (green area).

It is proposed that the C/S ratio should be at least 0.5. It follows that the distance to the center should also be less than ~0.5. This weakly depends on the ratio: instrument swath / radius of ocean structure as shown in figure 2.

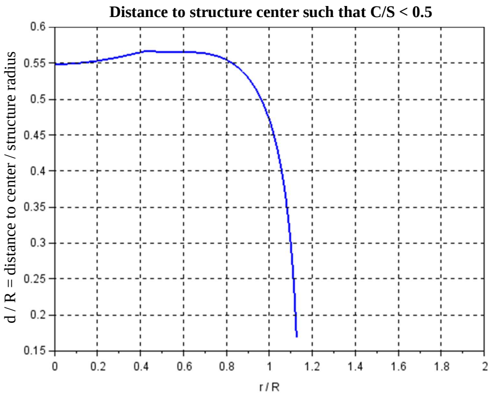


Figure 2: Determination of criterion for nadir altimeter

From now on, the value f = 0.5 will be assumed.

<u>Wide swath</u>:
This is much simpler and straightforward: the arc length covered at a given latitude is simply the length covered by the swath.

<u>Synthesis</u>:
Both instruments are treated in the same way through a pseudo swath whose dimension is F = 0.5*D (= 25 km) for nadir altimeters, F = 120 km for wide swath interferometers.

It can be noted that one wide swath instrument is theoretically equivalent to 5 nadir instruments (regarding the pseudo swath width only).

3.2. Method used for performance evaluation

The coverage performance index is based on a "longitude-time" map. These maps show all the passes at a given latitude. The computation period is 30 days. The longitude interval is chosen to be representative enough (30 degrees).

One example is shown in figure 3. Each ellipse represents a pass either ascending or descending. The ellipse dimensions along the x and y axes are the pseudo swath size and the expected revisit time respectively. The performance is measured by the coverage ratio: ratio of colored (covered) area: 75.3% in the example.

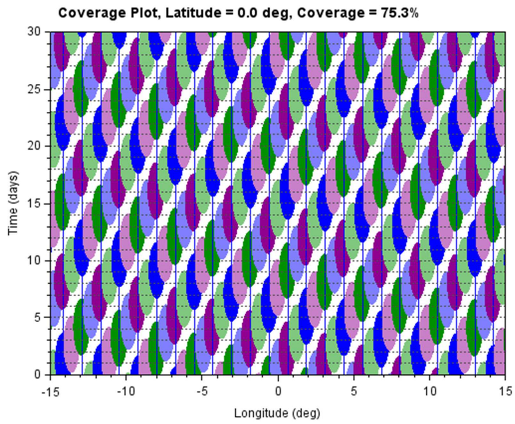


Figure 3: Coverage map at fixed altitude

The coverage ratio can be computed at all latitudes. Figure 4 shows an example of how coverage varies with latitude: depending on where ascending and descending passes intersect, the ratio can be locally minimum or maximum.

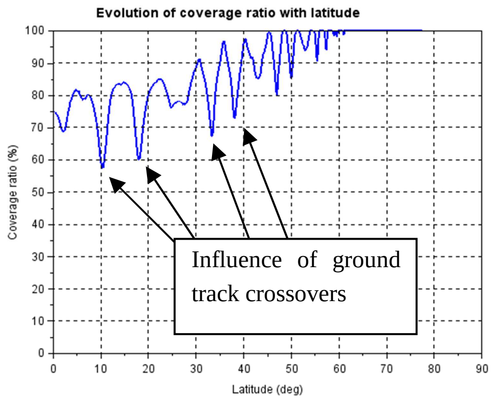


Figure 4: Coverage ratio as function of latitude

An empirical distribution function is then deduced (figure 5), where each individual coverage ratio is weighted according to the area involved (cosine of latitude).

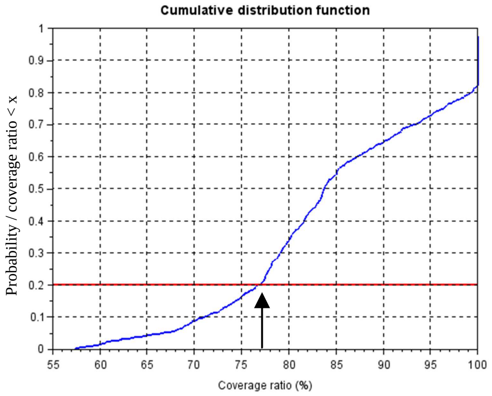


Figure 5: Coverage distribution function

The chosen criterion is to have a coverage ratio of (at least) 80% with a "probability" of (at least) 0.8. This means that the performance index (x-axis) such that the probability (ordinate) is 0.2 should be greater than 80%. We see that it is almost the case in the example.
It can also be noted that the 20% worst cases correspond to particular latitudes which are then simply ignored.

The criterion as defined so far is adapted to one well defined orbit. It can be generalized to several orbits or constellations. If some parameters are uncertain or not known they can be randomly chosen. That's the case for instance for the ascending nodes, the initial longitudes, the phasing between planes (gaps between argument of latitudes of reference satellites between planes), and even altitude. The relative distribution of arguments of latitude of satellites in the same plane is supposed known.

An example is given in figure 6. The constellation consists of 3 Sun-synchronous orbits (3 different orbit planes). Each plane contains 5 satellites at equally distributed argument of latitudes. The altitude is supposed fixed (597 km), but the phasing between the planes, or the longitudes or ascending nodes are not and are randomly chosen. Each dark blue curve corresponds to one particular choice.

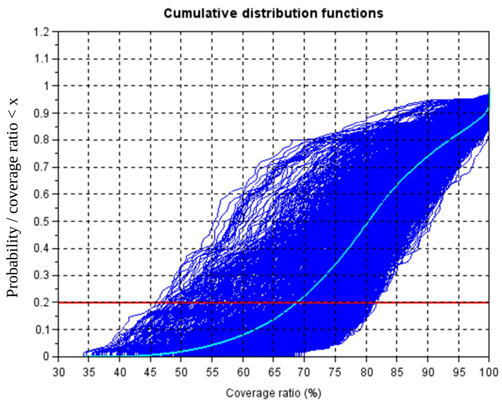


Figure 6: Distribution functions - several cases

The light blue curve is the distribution function considering all the results together: it gives a mean value (here 69%) which is the performance without precise knowledge of the orbits. Otherwise the maximum value can be used as a good enough estimate of the optimal performance (here: about 82%). There is of course some estimation uncertainty, but it is considered as acceptable.
In all cases, it should be checked that there is no long-term variation of the performance index as the computation is done over 30 days only.

4. Performance evaluation

The simplest constellations consist of 1 plane containing satellites at equally spaced arguments of latitude. That's why they are evaluated first. The orbits are Sun-synchronous.

When evaluating the performance in the altitude range 571-613 km and for a variable number of satellites, one gets the results shown in figure 7. The criterion is thus met for a constellation of at least 15 satellites.
The figure also shows that for a fixed number of satellites, all altitudes (i.e. repeat orbit parameters) are not equally good. Or conversely, there are cases where better results are obtained by a constellation consisting of fewer satellites.

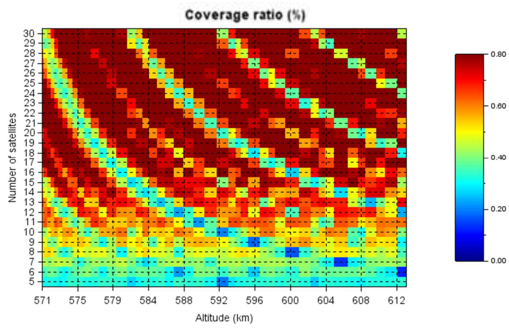


Figure 7: Coverage ratio (swath size = 25 km)

For wide swath instruments, the number of satellites is of course smaller: at least 3 is required, which is consistent with the fact that the swath is ~5 times larger than for nadir altimeters.

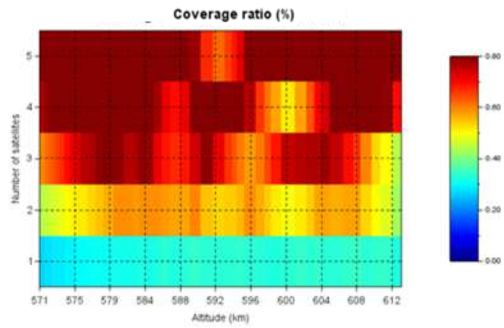


Figure 8: Coverage ratio (swath size = 120 km)

As a summary, the performance is met for 15 "nadir" or 3 "wide swath" satellites in a Sun-synchronous orbit at an adequate altitude.

5. Analysis

5.1. Simplified criterion

When considering ascending passes at latitude 0 only, one obtains for a one-plane (SSO) constellation the following result:

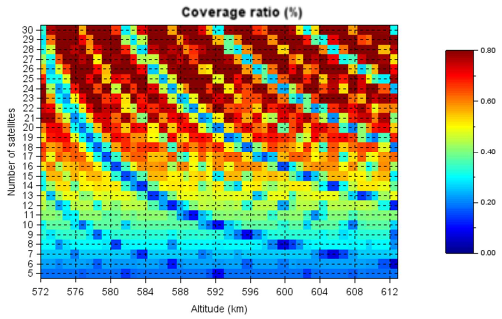


Figure 9: Coverage ratio at ascending nodes

This figure can be compared to figure 7. The performance indices differ but the maxima and minima correspond to the same altitudes and number of satellites.

Let's take a closer look by considering 2 cases:

- 15 evenly spaced satellites (swath = 25 km)
- 3 evenly spaced satellites (swath = 120 km).

Figures 10 and 11 show the performance index computed in 2 different ways:

- all latitudes (between 0 and 80 deg) and ascending and descending passes,
- latitude 0 and ascending passes only.

The performance index as function of altitude is shown in figures 10 and 11. The performance index for case 2 (ascending nodes only) has been empirically multiplied by 1.5 so that the 2 results could be more easily compared with each-other.

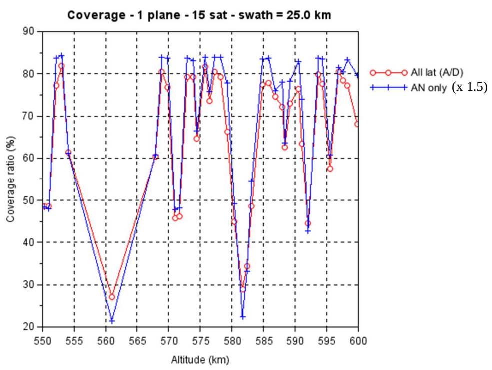


Figure 10: Performance index - 2 methods

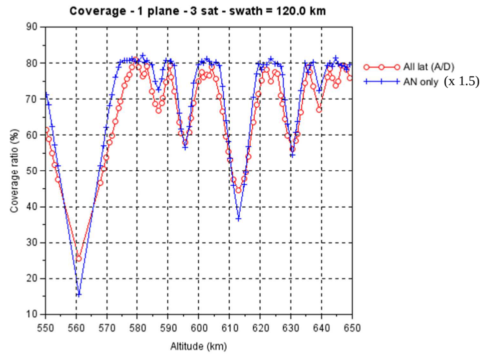


Figure 11: Performance index - 2 methods

The results considering all latitudes and ascending/descending passes are then similar to the ones considering ascending passes at the equator only: same favorable and unfavorable altitudes. It then makes sense to use the simplified criterion when computation time matters.

5.2. Choice of inclination

There are various factors that may influence the choice of inclination:

- the mission itself: the observation of polar zones for instance requires a high inclination,
- energy: SSO enable a better orientation of the solar arrays for example,
- aspects as the number of launches which should be minimized, so that one often prefers to have all satellites in the same plane,
- optimization of coverage.

In order to measure the gain of performance (if any) coming from various inclinations instead of only one, 2 cases are considered:

- Case 1: only one plane, inclination = 90 deg
- Case 2: 3 planes, inclinations = 30, 60 and 90 degrees.

A first conclusion is obtained using a simple criterion based on the number of passes only.

Let's call ΔL, the length of the intersection of the swath with a parallel (at some latitude). The proportion of longitude covered per orbit (2 passes) is then: p = 2 * ΔL / (2 * pi)

The objective here is that the parallel should be entirely covered within the expected revisit time, which means:

p * (Trev / T) = 1, where T is the orbit period.

One can then simply solve for the number of satellites in each plane by computing the performance index:

- at latitude 60 deg by including the contribution of plane 3 only (inclination = 90 deg),
- at latitude 30 deg by including the contributions of plane 2 and 3 (inclination = 60 and 90 deg resp.),
- at latitude 0 deg by including the contributions of planes 1, 2 and 3 (inclination = 30, 60 and 90 deg resp.).

which comes down to solving a simple 3x3 linear system:

$$\boxed{\begin{aligned} &n_3 * p_{3,3} = 1 \\ &n_2 * p_{2,2} + n_3 * p_{3,2} = 1 \\ &n_1 * p_{1,1} + n_2 * p_{2,1} + n_3 * p_{3,1} = 1 \end{aligned}}$$

where n_k is the number of satellite in plane k and $p_{i,j}$ is the covered longitude ratio by plane i at latitude j (latitude 1 = 0 deg, latitude 2 = 30 deg, latitude 3 = 60 deg).
Note that the previous calculation leads to satellite numbers that are not whole numbers.

The results for a swath size of 100 km and a revisit time of 5 days is:

Inclination	30 deg	60 deg	90 deg	Total
Number of satellites	1.0	3.6	5.9	10.5

Whereas with only one plane, the result is

Inclination	90 deg	Total
Number of satellites	11.7	11.7

The gain considering 3 planes (3 inclinations) instead of 1 is then around 10% on the total number of satellites, which is not so much.

Let's now come back to our "standard" criterion (coverage ratio of 80% with a "probability" of 0.8).

The criterion is met with a constellation of 15 (uniformly distributed) satellites in 1 plane (inclination = 90 deg). The performance index found is 82.5%.
Let's evaluate the second configuration consisting in 3 planes.

Because the distribution of satellites in the planes is uncertain (the previous estimation not be optimal), all possibilities with 15 satellites and at least 1 satellite per plane are tested: 91 cases in all.
The best performance was found for the case:

Inclination (deg)	90	60	30
Number of satellites	1	12	2

The performance index for this case is 84% > 82.5%.
This result is consistent with the simplified analysis, and confirms that there a little benefit of having several inclinations. However, the performance gain is limited.

5.3. Favorable altitudes (evenly distributed satellites)

In this part, we consider ascending passes at the ascending node only (see previous paragraphs for explanation).
The constellation made of one plane and the arguments of latitude are evenly spaced between 0 and 360 degrees. The number of satellites can be any number, but for demonstration purposes the illustrations will consider only 5.
The swath is chosen so that the performance is close to 80%.
The computation is done over the repeat period. All repeat orbits in the altitude range 500-1000 km with repeat periods up to 30 days are evaluated.
Figure 12 shows the performance index as function of altitude. The performance is almost periodic with favorable altitudes and less favorable ones.

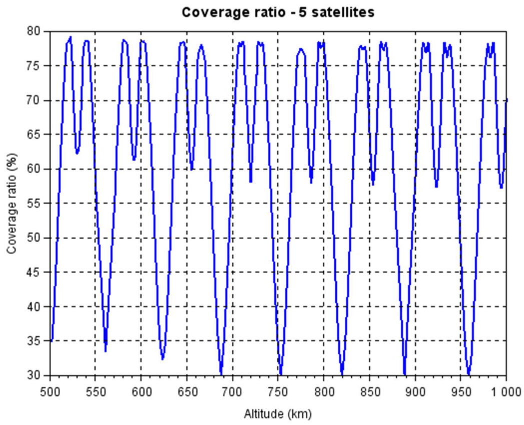


Figure 12: Coverage ratio (function of altitude)

The orbits where the performance index is minimum are such that: P / Q = k / nbsat,
where P and Q are the phasing parameters, nbsat is the number of satellites, and k is a whole number between 0 and nbsat. This is more clearly shown in figure 13 where the performance index is plotted as function of nbsat * (N + P/Q). The performance is minimum when the abscissa is a whole number, and it is degraded when the abscissa is a whole number + 0.5.

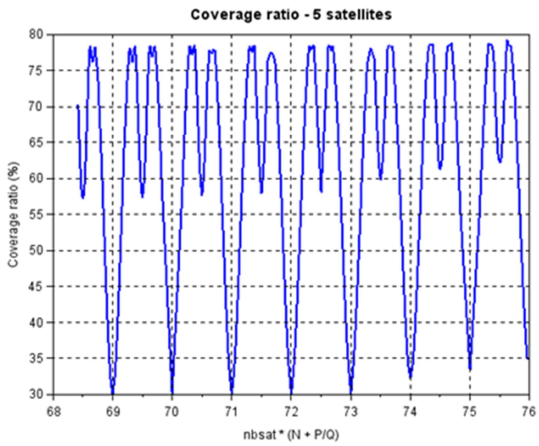


Figure 13: Coverage ratio

The explanation is rather simple. The ellipses in the longitude-time plots intersect more or less depending on the chosen repeat parameters. If P / Q = k / nbsat, the satellites are in the same ground track.
To illustrate this aspect, two altitudes are considered: 690 km and 709 km (see figures 14 and 15). For the first (unfavorable) one, the ellipses intersect a lot whereas for the second (favorable) case, the ellipses are well distributed in the plane, which maximizes the coverage.
Note that the x and y axes are normalized: x represents the longitude (the interval represented corresponds to the difference of longitude between 2 consecutive passes of 1 satellite), y represents time (one repeat period).

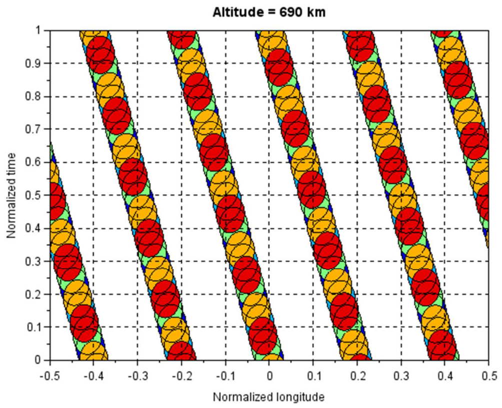


Figure 14: Coverage map (low performance)

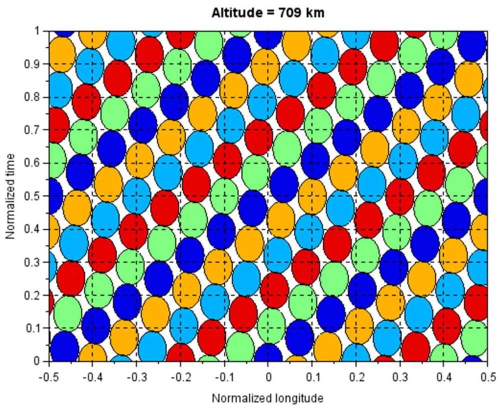


Figure 15: Coverage map (high performance)

5.4. Optimization of geometry

A natural (and simple) choice consists in choosing evenly distributed arguments of latitudes for satellites in the same plane. But that's not necessarily optimal as shown in the previous sections.

Two methods can then be considered to handle this difficulty:

1) use evenly space arguments of latitude, but consider altitude as a "free" parameter (and allow it to vary in a large enough range),
2) find an optimal distribution of the arguments of latitude for satellites in the same plane.

Both methods are found to be (almost) equivalent, provided the altitude range for method 1 is large enough.

In figure 10, we see that altitude 582 km is not favorable for 15 equally distributed satellites: the performance index is about 29%.

Using a simple optimization algorithm (global search), the performance is found to be 82.1%, which is close to the best value found for any altitude.

Of course Method 2 is time consuming if the nominal criterion is used. A more efficient way is to use the simplified criterion (coverage of ascending passes at the equator) to find optimal arguments of latitude from which the performance index can be computed. Using this method, the value found is 81.4% which is very close to the previous one.

But this may not work in all cases, in particular for any number of satellites. In figure 16, performance indices using evenly spaced and optimized satellite positions are compared. The criterion is the simplified one, and the constellation is made of 3 satellites. In addition, the swath size has been adjusted for the coverage ratio to be close to 80%. We see that the performance for the optimized geometry is almost optimal, but there are cases when the performance index cannot be improved as around the altitude corresponding to specific repeat periods.

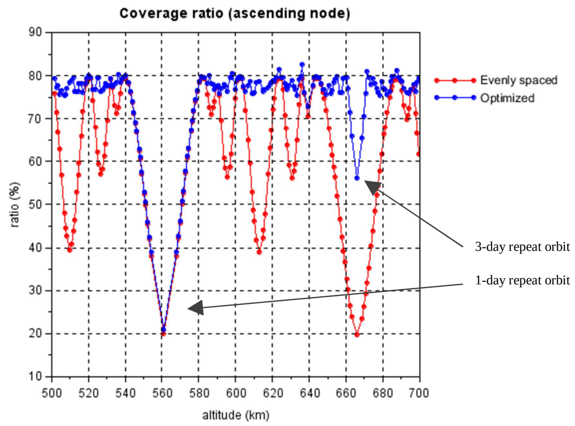


Figure 16: Long-term evolution of performance index

6. Evaluation for heterogeneous constellations

The objective here is to evaluate the performance of heterogeneous constellations in a realistic situation. The constellations are made of polar, JASON-like and Sun-synchronous orbits. There is only one JASON-like orbit, only one polar orbit, and up to 4 SSO (with all satellites in the same plane).

Two methods are used which give similar results:

1) The SSO satellites are assumed evenly spaced in the plane and the altitude is unknown in the range 571-613 km.
2) The altitude is fixed (814 km), and the geometry is first optimized.

The performance is obtained from 200 cases where all uncertain parameters are randomly drawn: initial argument of latitude, longitude, local time of ascending node and altitude (for method 1).

The results found are the following:

Configuration	Coverage - min / max (optimized geometry) (%)
1 P + 1 J + 3 S	31.7 / 32.3
1 P + 1 J + 2 S + 1 W	51.9 / 52.6
1 P + 1 J + 1 S + 2 W	65.8 / 73.3
1 P + 1 J + 3 W	81.0 / 81.6
1 P + 1 J + 4 S	38.0 / 38.5
1 P + 1 J + 2 S + 2 W	72.2 / 72.8
1 P + 1 J + 1 S + 3 W	83.5 / 84.5
1 P + 1 J + 4 W	92.0 / 92.6

S means "nadir satellite" (swath size = 25 km), W means "wide swath" (swath size = 120 km).
The performance with only nadir instruments is limited: around 30% for 5 satellites. And of course it is much improved when large swath instruments are added.

Figure 17 shows the long-term evolution of the performance index, and as a matter of fact there are only limited variations.

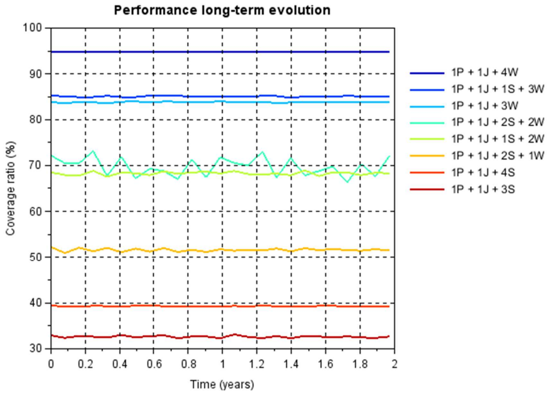


Figure 17: Long-term evolution of performance index

The constellations that were found appear satisfactory.

7. Conclusion

The paper has shown the criteria and methods used to assess the performance of altimetry constellations. The methods proved efficient without the need for complex optimization tools. This due to the fact that it is generally possible to choose regularly spaced arguments of latitude for satellites in the same plane provided altitude is adequately chosen. And because the performance is not much improved by adding planes with low inclinations, the number of planes can be reduced.
The criterion as defined is in fact not specific to altimetry missions and could be used for other applications for which coverage is a key aspect.

All computations were done using Scilab and CelestLab space mechanics toolbox [2].

References

1) *The next 15 years of satellite Altimetry - P Escudier (CLS), JL Fellous (orange Bleue Conseil), CEOS-Noaa-Eumetsat*

2) *CelestLab, space mechanics toolbox for Scilab, https://atoms.scilab.org/toolboxes/celestlab*