Reducing the Lunisolar Numerical Ephemerides for Onboard Applications

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Lunisolar ephemerides are necessary for orbit computation, however, the ephemeris parameterization can sometimes be a problem for onboard applications. On one hand, onboard computation requires that the ephemeris parameterization should be concise, as a result, the off-the-shelf numerical ephemeris such as the JPL DE series are too large in size to fit in; on the other hand, the onboard application sometimes requires accurate lunisolar ephemerides, whereas the mean secular orbit from analytical theory becomes insufficient in accuracy. Here in this paper, we report two approaches to remodel the lunisolar ephemerides. To keep higher accuracy than the analytical mean orbit, both approaches start with the JPL DE ephemeris. We show that the new parameterizations take less storage than the off-the-shelf DE ephemeris while exceed the analytical mean orbit in accuracy.

Key Words: Lunisolar Ephemeris, Size Reduction, Onboard Application

Nomenclature

T_i	:	the i^{th} period in FFT
s_i	:	amplitude of the i^{th} sine component
c_i	:	amplitude of the i^{th} cosine component
Δt	:	elapsed time since initial/reference epoch
μ	:	geocentric gravitational constant
$\vec{r}$	:	geocentric position vector
$\hat{r}$	:	unit vector of $\vec{r}$
$\ddot{\vec{r}}$	:	geocentric acceleration
δx	:	error/deviation of variable x
Superscripts		
$'$ (prime)	:	counterpart variables for perturbing body (e.g. seleno-/heliocentric gravitational constant or position vector)

1. Introduction

The lunisolar ephemerides are essential parts in orbit computation, in terms that they are needed to compute the perturbing accelerations of lunisolar gravitations, tidal forces, solar radiation pressure and so on. In many cases, this is not a major concern, since the lunisolar ephemerides can be readily computed with a variety of methods. The analytical solutions to the lunisolar orbit motion, which result from the conventional celestial mechanics, is often a simple and convenient option when the low or moderate accuracy is needed. In such occasions, secular orbits that describe the mean long-term variation of the lunisolar orbits (sometimes also including major period terms) and have simple forms that are always welcome in orbit computations.[1)] When the higher accuracy is required than the analytical secular orbits, various numerical ephemerides are also conveniently available, such as the JPL DE series. These models are very accurate but take large storage since they are exhaustive for all the large planets and the Sun. DE series also include lunar libration and many other constants.

However, when the orbit is to be computed onboard satellites, the ephemeris parameterization can sometimes be a problem. On one hand, onboard computation requires that the ephemeris parameterization should be concise, so the off-the-shelf numerical ephemeris may be too large in size to fit in; on the other hand, the onboard application sometimes requires accurate lunisolar ephemerides, whereas the mean secular orbit from analytical theory becomes insufficient in accuracy. Complex lunar motion theories[2,3)] that dig deeper into the dynamics could provide better analytical solution, but they are always too complicated to be practically applied in onboard space missions. Semi-analytical lunar theories are also developed (see e.g. Ref 4, 5)), which combine the analytical expansions of various forms and the accurate measurements or numerical ephemeris. The solution could have matching accuracy with numerical ephemeris, but at the cost of 10^4-10^5 terms for each component (longitude, latitude or distance).

What is often needed in space missions is a trade-off between the numerical ephemeris and the pure or semi-analytical solution, and is expected to have a concise form with moderate accuracy. A common approach is to reanalyze the ephemeris with Fourier transformation, where major frequencies can be revealed (see e.g. Ref 6)). The complexity of selected frequencies, which is generally consistent with the accuracy of the fitted equation, reflects the storage taken by the remodeled parameterization.

Besides the fitting approach, another more straightforward approach used in this work is to directly remove the irrelevant parameters in the original numerical ephemeris. For the orbit computation, the lunisolar orbits, or sometimes only the lunar orbit, are critical. So for the major planets, it is possible that the secular mean orbits[7,8)] are already sufficient, while the respective parameters in the numerical ephemeris can be safely removed.

The work in this paper is set in the background of orbit prediction (OP) onboard the GEO satellite. The onboard OP is expected to be responsible for the orbit of the next 4 hours with reasonable accuracy. The model parameters are designed to be updated every 180 days, suggesting that if the ephemerides are to be simplified, each set of simplified ephemeris parameters should satisfy the accuracy within the period.

The paper is organized as follows. The fitting approach is discussed in Section 2 and the approach of tailoring the ephemeris is explained in Section 3. In Section 4, the implication of lunisolar ephemeris error on orbit prediction is discussed. Some final discussions are given in Section 5.

2. Fitting the Ephemeris with Frequency Analysis

Given the requirement explained in the introduction, the following strategies are taken to process the ephemeris:

- The DE406 ephemeris is chosen. Other ephemerides can also be used, while the following procedures can be likewise applied;
- The Cartesian coordinates are fitted, since they are directly needed to compute the perturbing acceleration;
- The ephemeris with the length of about 180 years is analyzed, for either the Moon or the Sun, to find out the essential frequencies. During fitting, the frequencies are consistent and only the bias, trend and amplitudes are fitted per 180-day segment;
- To better reveal the high frequencies, the ephemerides are extracted 10 points per day.

For each component of the Cartesian states, the fitting function takes the following form

$$x(t) = a + b \cdot \Delta t + \sum_{i=1}^{N} \left[s_i \sin\left(\frac{2\pi}{T_i}\Delta t\right) + c_i \cos\left(\frac{2\pi}{T_i}\Delta t\right)\right], \quad (1)$$

where a and b are fitted bias and linear trend. The period T_i (or equivalently the frequency $f_i = 2\pi/T_i$) are determined from spectral analysis of the 180-year long ephemeris.

Our practice of fast Fourier transformation (FFT) shows that some signals are small or even buried in the noises. Although these small signals are difficult to be identified in the spectrum, they can be obviously found in the fitting residuals. Therefore, it is not enough to simply analyze the time series of the Cartesian states once.

In practice, the fitting procedure and residual analysis are repeated until the residuals no longer represent obvious periods. This requires manual intervention when choosing or adjusting the frequencies. *No obvious periods* in the residuals means that if the better accuracy is pursued, further frequencies to be included would be much more complicated than a few individuals. This is the case for the lunar orbit, which is strongly perturbed by the Sun and the large planets.

The processing of the lunisolar ephemeris based on the DE406 model shows that about 17 and 5 evident frequencies can be found in the lunar and solar orbits respectively, both of which are shown in Fig. 1. The frequencies for each component are close but the numeric values may not be exactly the same.

To make sure that the found frequencies are consistent, multiple tests are computed to check the consistency and accuracy. Twenty segments of 180-day long ephemeris are fitted, using the found frequencies.

Fig. 2 shows the fitted amplitudes of the position components. Most fitted amplitudes of these frequencies are consistent in different tests and do not show significant discrepancy.

More importantly, the remodeled ephemerides using the fitted Eq. (1) match well with the original numerical ephemerides. Fig. 3 shows the 3D position error between the ephemeris and

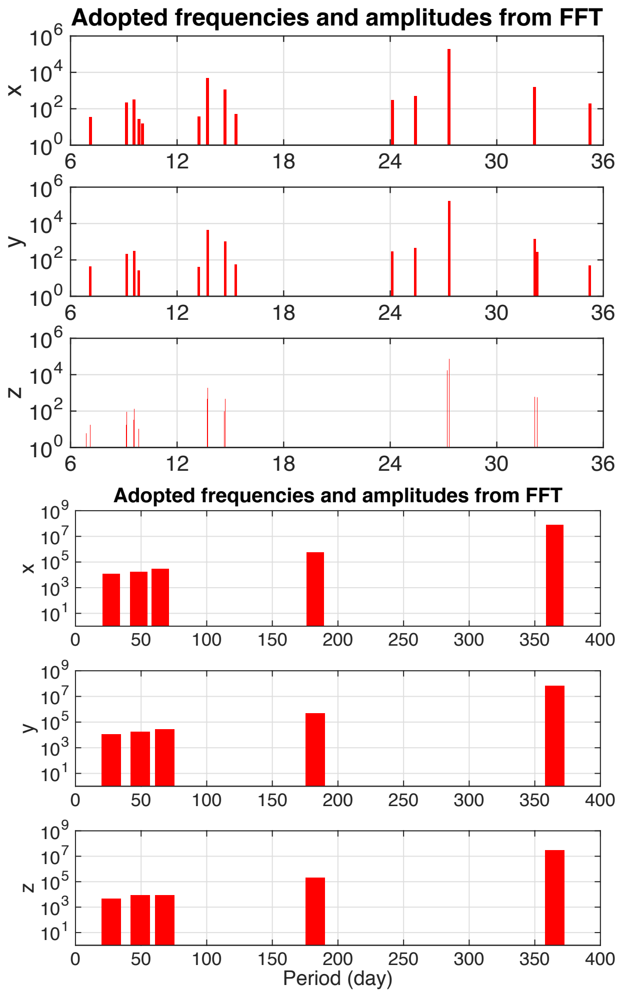

Fig. 1. Frequencies selected for following fitting, for three position components of both the Moon (top) and the Sun (bottom).

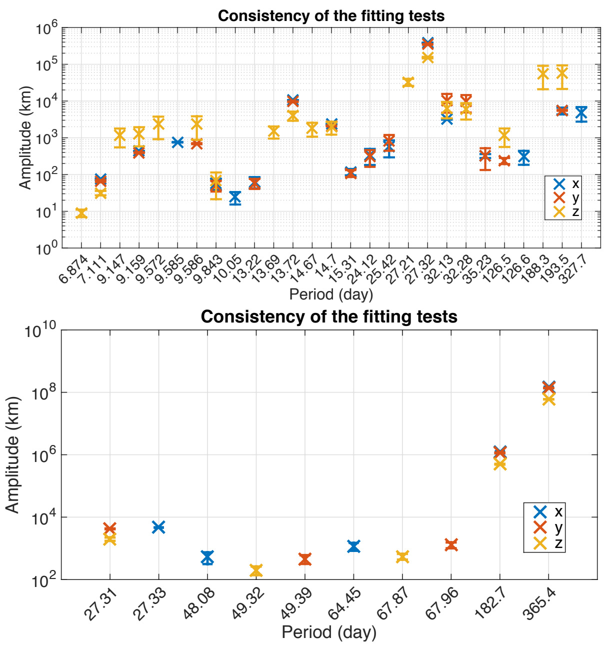

Fig. 2. Fitted amplitude of all three directions, for both the Moon (up) and the Sun (bottom). The error bar suggests the standard deviation of all the fitted amplitudes, with respect to the mean value.

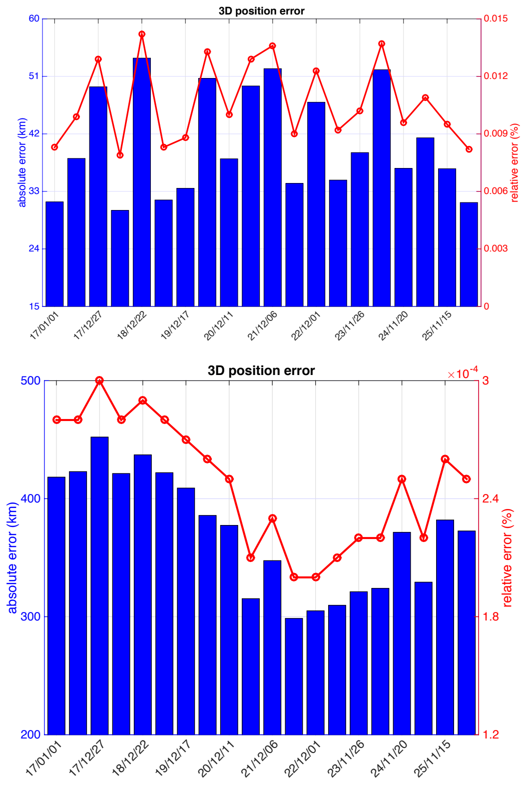

Fig. 3. Error of the fitted position with respect to the ephemeris values, for the Moon (up) and the Sun (bottom). The left y-axis, corresponding to the blue bars, is the absolute error (in *km*), while the right y-axis, corresponding to the red lines, is the relative error (in percentage) of the absolute error with respect to the true distance at the epoch.

fitted values. For all tests, the relative fitting errors are consistent, of about 10^{-4} for the Moon and 10^{-6} for the Sun, which are well better than the mean secular orbit which are no better than 10^{-2} for the Moon and 10^{-3} for the Sun.

The new parameterization would have an implication on acceleration, which further affects the OP accuracy. The error of the lunisolar ephemeris on the acceleration is to be discussed later.

3. Tailoring the Ephemeris

In contrast to the analytical formulation of the lunisolar orbit, the numerical ephemeris allows easy access to the accurate lunisolar ephemerides. However, the off-the-shelf parameterization, even compiled in compact binary format, is still too large to be applied in onboard application. Nevertheless given the fact that ephemerides are exhaustive for all the major planets and that the accuracy is well beyond the requirement of many onboard applications, it is possible that the ephemerides be reduced by removing excess parameters and by trading the appropriately reduced accuracy with reduced data size.

A first motivation is that the accurate planetary ephemerides are not necessarily needed. Since the perturbing accelerations of the major planets are very small, only lunisolar ephemerides need to be accurately considered while the analytical formulas can be applied for the planets.[7,8] In some cases, only lunar ephemeris is of major concern, while the secular solar orbit might already be sufficient.

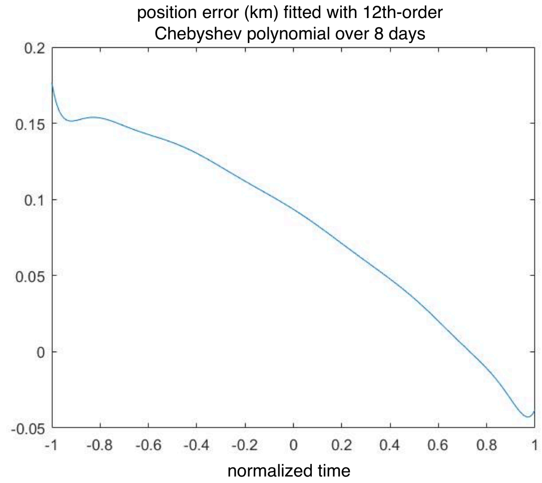

Fig. 4. Error of re-fitted x component of lunar position (in *km*), fitted over 8 days using 12th-order Chebyshev polynomial.

The *lossless* reduction can be implemented as follows. Various ephemerides always cover centuries to millennia, but the lifetime of a typical Earth satellite only spans 10 to 20 years. Reduction procedure starts with choosing the time of interest. Every data file of DE405 covers 20 years (varies among different versions) and in each file the data blocks are organized chronologically. In each data blocks, the coefficients of the Chebyshev polynomials are stored in unique order, with which we can simply extract the lunisolar parameters and leave the planetary/nutation parameters.

When the appropriate parameters are available, they can be likewise compiled in binary format, using the JPL official code with moderate revisions to the parameter number, start/end point in the data block and the number of celestial bodies. After this first reduction, the lunisolar ephemerides remain lossless in accuracy.

Tests show that for one data file of DE405, which covers 20 years and contains 229 data blocks, the original binary file takes 6254 kilobytes (KB) of storage while the reduced binary file with lossless lunisolar ephemerides only takes 827 KB of storage. It suggests an approximately $1/8 \sim 1/7$ reduction rate although the value may vary among different versions.

Further reduction can be obtained if appropriate loss of accuracy is acceptable. Acknowledging the fact that the numerical ephemeris is represented in the coefficients of 12th-order Chebyshev polynomials and that the ephemerides are fitted over certain time lengths specific for individual bodies, we can try to fit the ephemerides with lower order of Chebyshev polynomials and/or over longer time spans.

Taking the lunar ephemeris as an example, due to its fast motion with respect to other celestial bodies, the lunar ephemeris is fitted every 4 days in DE405 (in contrast to 32 days for many other planets). Fig. 4 shows the error of the re-fitted x component of lunar position in an arbitrarily selected 8-day segment. The 8-day series x components computed using original DE405 ephemeris, supposedly from two 4-day segments, are re-fitted using one 12th-order Chebyshev polynomial. The re-fitted ephemeris, which is only half the original size, has a maximum error of less than 200 meters.

On the other hand, the fitting procedure can also be performed using lower order Chebyshev polynomials, in pursuit of

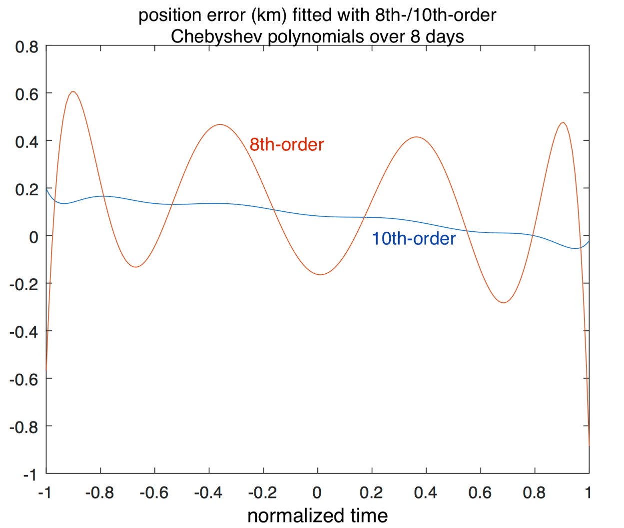

Fig. 5. Error of re-fitted x component of lunar position (in km), fitted over 8 days using 8th-order (red) and 10th-order (blue) Chebyshev polynomials.

less parameters. Fig. 5 shows the errors of the re-fitted x component over 8 days but using 8th-order and 10th-order Chebyshev polynomials respectively. As expected, fitting with lower order increases the fitting error, but it is still well better than $1km$. Compared with the initial *lossless* reduction, this reduces the parameter numbers by over 60 percent but outperforms the fitting approach (frequency analysis) by $1 \sim 2$ order of magnitude in accuracy.

For a semi-annual ($\sim$ 180 days) segment, we can expect approximately $(800/40) \times 40\% = 8$ KB in binary data storage. This value can still be reduced if the fitting span and the polynomial order are further adjusted.

4. Implication on the Acceleration

Given the perturbing body is much farther than the satellite from the Earth, the perturbing acceleration can be approximated as

$$\ddot{\vec{r}} \approx \frac{\mu' r}{r'^3} \left(\hat{r} - 3\cos\Psi \hat{r}'\right) , \tag{2}$$

where Ψ is the geocentric elongation between the Earth and the perturbing body. It can be easily proved that

$$||\ddot{\vec{r}}|| \leq 2 \cdot \frac{\mu' r}{r'^3} . \tag{3}$$

Normalizing the acceleration with respect to the two-body gravitation (which benefits further dynamic analysis), the implication of the position error of the perturbing body on the acceleration reads as follows

$$\delta||\ddot{\vec{r}}||_{\max} = 6\left(\frac{\mu'}{\mu}\right)\left(\frac{r}{r'}\right)^3\left(\frac{\delta r'}{r'}\right) . \tag{4}$$

The acceleration error is plotted in Fig. 6, against the geocentric distance. It is clear that the solar ephemeris can be easily fitted. With 5 frequencies in each direction, the fitted acceleration is below 0.4×10^{-10} for MEO and below 1.4×10^{-10} for GEO and IGSO. It is sufficient for orbit propagation over a few hours to reach cm-level accuracy.

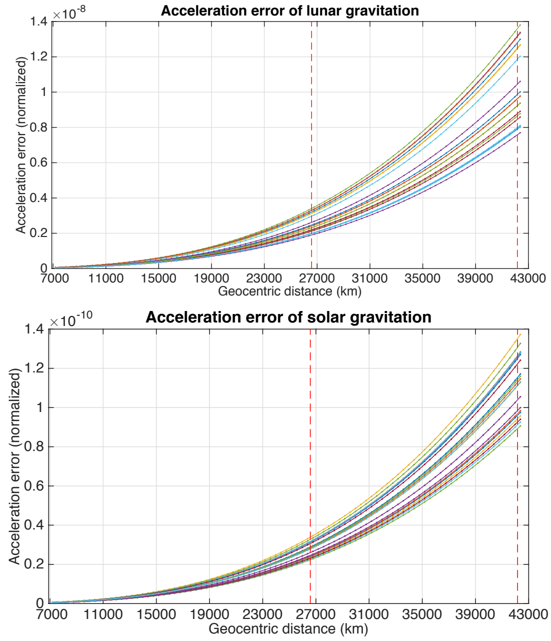

Fig. 6. The normalized acceleration error with respect to the orbit altitude, for the Moon (up) and the Sun (bottom). Various lines correspond to the fitted position errors in Fig. 3. The two vertical lines show the altitudes of MEO and GEO/IGSO.

However, for lunar ephemeris, the acceleration error can be as large as 10^{-8} at GEO altitude. Such error is not always negligible, especially for accurate applications. On the other hand, alternative approach, such as tailoring the numerical ephemerides, allows accurate approximation. Based on the approaches in Section 3, the ephemerides can be further approximated if large errors are acceptable. This would allow for better trade-off between ephemeris accuracy and data size.

5. Conclusion

In this paper, we have shown our preliminary results on two approaches to reduce the lunisolar ephemerides for our specific onboard application. The frequency analysis allows simple modeling of lunisolar ephemerides over 180 days with less than 50 parameters, however, the accuracy of the remodeled lunar ephemeris may be insufficient for precision applications. Tailoring the discrete numerical ephemerides, on the other hand, allows *controllable* accuracy by gradually lowering the polynomial order and/or increasing the fitting span, although the coefficients of the Chebyshev polynomials normally take more storage than the frequency analysis. These two approaches can be alternatively used in onboard application regarding actual accuracy requirement.

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