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Adaptive Attitude Tracking Control with Parameter Convergence in the Absence of Persistent Excitation

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Abstract

A novel adaptive controller for attitude tracking control problems of rigid bodies is presented in this paper. The most important feature is that both instantaneous state data and past measurements (historical data) are concurrently employed during the parameter adaptation process. A specially designed information matrix is introduced to encode concurrent information into the adaptive law. Under this new formulation, both state tracking errors and parameter estimation errors are guaranteed to asymptotically converge to zero subject to the satisfaction of a finite excitation condition, which is a significant relaxation when compared to the persistent excitation condition that is typically required for these classes of problems. Numerical simulations are illustrated to evaluate the various features of the proposed method.

Keywords: Attitude Control, Adaptive Control, Finite Excitation

Introduction

Attitude control problems of rigid bodies have been extensively investigated in the past several decades due to their applications in aerospace engineering and many other areas such as robotics. In particular, nonlinear adaptive control methods have been widely studied and applied to the stabilization/tracking control problems of attitude dynamics in the presence of parameter uncertainties. It is important to note that most of the existing adaptive attitude control solutions are built upon the conventional certainty equivalence (CE) principle. It is well understood that CE-based adaptive controllers cannot guarantee the convergence of parameter estimation errors to zero unless reference signals additionally satisfy certain persistent excitation (PE) conditions [1]. This fact further results in potential performance degradation of CE-based adaptive controllers in many applications [2] when compared with the underlying deterministic controllers (i.e., controllers for non-uncertainties system dynamics).

Aiming to address the possible absence of persistence of excitation, the main contribution of this paper is the introduction of a new adaptive control scheme for attitude tracking control problems of rigid bodies in the presence of parameter uncertainties. It should be emphasized that the results presented in this paper are partially inspired by the concurrent learning adaptive control (CLAC) theory [3, 4], but maintain certain crucial distinctions. The CLAC design innovatively uses specially selected and online recorded state data concurrently with instantaneous state data for adaptation. Under the CLAC framework, if system states can be assumed to satisfy a certain sufficient excitation condition over a finite interval of time (which is formally referred to as a finite excitation (FE) condition), the adaptive control algorithm can be designed such that rich enough historical data could be recorded to ensure the convergence of parameter estimation errors. However, to acquire the historical data, smoothers or observers need to be employed to numerically approximate state derivatives, a process that is usually vulnerable to multiple sources of measurement noise and approximation errors. This requirement for smoothers also lays a great theoretical barrier to the applications of CLAC to nonlinear mechanical systems due to the inevitable coupling that exists between state derivatives and unknown parameters. In particular, it is currently impossible to design a concurrent learning adaptive controller for attitude control problems.

In this paper, a novel adaptive tracking control algorithm ensuring precise parameter estimation under a FE condition is developed for attitude tracking control problems of rigid bodies. To be specific, low-pass filtered regressor matrices and states are first introduced into the formulation, which not only circumvents the state derivative estimation requirements of the classical CLAC formulation within the adaptation scheme but also renders the resulting parameter-adaptation dynamics to reside within a stable and attracting manifold. Subsequently, a special information matrix is designed to continuously record historical data and provide new information for the adaptation process. Additionally, a judiciously designed non-CE term is introduced into the adaptive algorithm to help the resulting closed-loop system overcome the uniform detectability obstacle [5] if the FE condition cannot be satisfied. Under this design framework, system states are ensured to asymptotically track the desired trajectories, and if system states further satisfy a FE condition, parameter estimation errors are further guaranteed to asymptotically converge to zero.

The remainder of this paper is organized as follows. The governing attitude tracking dynamics and the control objective are introduced in Sec. II. Then, the main result of the paper, a composite adaptive control algorithm is presented in Sec. III, along with a proof for the major closed-loop stability result. Numerical simulation results are demonstrated in Sec. IV. Finally, this paper ends with some concluding remarks in Sec. V.

Mathematical Preliminaries and Problem Formulation

Mathematical Preliminaries

Definition 1 (Finite Excitation) [3]: A bounded signal $g(\cdot):\mathbb{R}\to\mathbb{R}^{n\times m}$ is said to be finite exciting (FE) over an interval $[t,t+T]$, where $t\geq 0$ is finite, if there exist finite constants $T>0$ and $c>0$ such that

$$\int_t^{t+T} g^{\mathrm{T}}(\tau)g(\tau)\mathrm{d}\tau \geq c\mathbf{I}_{m\times m}$$

where $\mathbf{I}_{m\times m}$ is the m-dimensional identity matrix.

Problem Formulation

The unit-quaternion-based attitude kinematics is utilized in this paper. The body-fixed frame of the rigid body is denoted by $\mathcal{F}_b=\{X_b,Y_b,Z_b\}$, and the inertial frame is represented by $\mathcal{F}_i=\{X_i,Y_i,Z_i\}$. As an example, the unit quaternion of $\mathcal{F}_b$ concerning $\mathcal{F}_i$ is defined by [6]: $\boldsymbol{q}_{bi}=[\eta_{bi},\boldsymbol{\xi}_{bi}^{\mathrm{T}}]^{\mathrm{T}}$, where η_{bi} and $\boldsymbol{\xi}_{bi}$ are respectively the scalar part and the vector part of $\boldsymbol{q}_{bi}$, and $\eta_{bi}^2+\boldsymbol{\xi}_{bi}^{\mathrm{T}}\boldsymbol{\xi}_{bi}=1$. Multiplication is an important operation of quaternions; for arbitrary two quaternions $\boldsymbol{q}_1=[\eta_1,\xi_1^{\mathrm{T}}]^{\mathrm{T}}$ and $\boldsymbol{q}_2=[\eta_2,\xi_2^{\mathrm{T}}]^{\mathrm{T}}$, using `` $\otimes$ ” to denote the quaternion multiplication operator, we have $\boldsymbol{q}_1\otimes\boldsymbol{q}_2=[\eta_1\eta_2-\xi_1^{\mathrm{T}}\xi_2,(\eta_1\xi_2+\eta_2\xi_1+\xi_1\times\xi_2)^{\mathrm{T}}]^{\mathrm{T}}$. Moreover, $\boldsymbol{q}^*=[\eta,\xi^{\mathrm{T}}]^{\mathrm{T}}$ denotes the conjugate of the corresponding quaternion $\boldsymbol{q}$. For the tracking control problems considered in this paper, we employ a virtual frame $\mathcal{F}_t=\{X_t,Y_t,Z_t\}$ to describe the expected attitude trajectories and use $\boldsymbol{q}_{ti}$ to denote the quaternion of $\mathcal{F}_t$ with respect to $\mathcal{F}_i$. The error quaternion, which describes the relative attitude of $\mathcal{F}_b$ with respect to $\mathcal{F}_t$, is defined by $\boldsymbol{q}_{bt} = \boldsymbol{q}_{ti}^* \otimes \boldsymbol{q}_{bi}$, and the attitude-tracking error dynamics model is given as follows [6],

$$\dot{\boldsymbol{q}}_{bt} = \frac{1}{2} E(\boldsymbol{q}_{bt})\boldsymbol{\omega}_{bt}^b, \quad E(\boldsymbol{q}_{bt}) = \begin{bmatrix} -\boldsymbol{\xi}_{bt}^{\mathrm{T}} \\ \eta_{bt}\mathbf{I}_{3\times3} + S(\boldsymbol{\xi}_{bt}) \end{bmatrix} \tag{1}$$

$$\boldsymbol{J}\dot{\boldsymbol{\omega}}_{bt}^b = -S(\boldsymbol{\omega}_{bi}^b)(\boldsymbol{J}\boldsymbol{\omega}_{bi}^b) + \boldsymbol{J}[S(\boldsymbol{\omega}_{bt}^b)\boldsymbol{\omega}_{ti}^b - C(\boldsymbol{q}_{bt})\dot{\boldsymbol{\omega}}_{ti}^t] + \boldsymbol{u}_c \tag{2}$$

where $C(\boldsymbol{q}) = \mathbf{I}_{3\times3} - 2\eta S(\xi) + 2S^2(\xi)$, η_{bt} and $\boldsymbol{\xi}_{bt}$ are respectively the scalar part and the vector part of $\boldsymbol{q}_{bt}$. The vector $\boldsymbol{\omega}_{bt}^b$ denotes the relative angular velocity between the two frames, which is expressed in $\mathcal{F}_b$. We use $\boldsymbol{\omega}_{ti}$ to denote the expected angular velocity, and $\boldsymbol{\omega}_{ti}^b$ and $\boldsymbol{\omega}_{ti}^t$ denote the coordinate transformation of that same vector in $\mathcal{F}_b$ and $\mathcal{F}_t$, respectively. The matrix $\boldsymbol{J}$ is symmetric positive-definite designating the inertia of the rigid body, and $\boldsymbol{u}_c$ denotes the control input torque signal to be designed. Then the tracking control objective is to design $\boldsymbol{u}_c$, such that the closed-loop system is stable, and the rigid body can track the expected attitude trajectory (formalized as $\lim_{t\to\infty}\{\boldsymbol{\xi}_{bt}(t),\ \boldsymbol{\omega}_{bt}(t)\} = \mathbf{0}_3$) for the case of uncertainty within the inertia matrix $\boldsymbol{J}$ (unknown constant).

Composite Adaptive Controller Development and Stability Analysis

An important property of attitude-tracking dynamics is that it allows affine representation of inertia-related terms. In particular, this property is utilized to re-organize Eq. (2) to the following form:

$$\boldsymbol{J}\dot{\boldsymbol{\omega}}_{bt}^b = -k_p(\dot{\boldsymbol{\xi}}_{bt} + \kappa\boldsymbol{\xi}_{bt}) - k_d\boldsymbol{\omega}_{bt}^b + \boldsymbol{J}^{-1}(\boldsymbol{W}\boldsymbol{\theta} + \boldsymbol{u}_c) \tag{3}$$

where k_p, k_d and κ are positive constants, and $\kappa = k_p + k_d$; $\boldsymbol{\theta} = [J_{11}, J_{12}, J_{13}, J_{22}, J_{23}, J_{33}]^{\mathrm{T}}$, it contains all the information of the unknown inertia matrix $\boldsymbol{J}$, wherein J_{11}, J_{12}, J_{13}, J_{22}, J_{23} and J_{33} are entries of $\boldsymbol{J}$; $\boldsymbol{W}$ is a regressor matrix which satisfies $\boldsymbol{W}\boldsymbol{\theta} = \boldsymbol{J}[k_p(\dot{\boldsymbol{\xi}}_{bt} + \kappa\boldsymbol{\xi}_{bt}) + k_v\boldsymbol{\omega}_{bt}^b + S(\boldsymbol{\omega}_{bt}^b)C(\boldsymbol{q}_{bt})\boldsymbol{\omega}_{ti}^t - C(\boldsymbol{q}_{bt})\dot{\boldsymbol{\omega}}_{ti}^t] - S(\boldsymbol{\omega}_{bi}^b)(\boldsymbol{J}\boldsymbol{\omega}_{bi}^b)$. Then, we define the following filtered angular velocity and regressor matrix,

$$\dot{\boldsymbol{\omega}}_f = -\kappa\boldsymbol{\omega}_f + \boldsymbol{\omega}_{bt}^b, \quad \omega_f(0) = \mathbf{0}_3 \tag{4}$$

$$\dot{\boldsymbol{W}}_f = -\kappa\boldsymbol{W}_f + \boldsymbol{W}, \quad \boldsymbol{W}_f(0) = \mathbf{0}_3 \tag{5}$$

and also employ an auxiliary control input $\dot{\boldsymbol{u}}_f = -\kappa u_f + u_c$. Substitute $\boldsymbol{\omega}_f$, $\boldsymbol{W}_f$ and $\boldsymbol{u}_f$ into Eq. (3) and by straightforward algebraic operations, one has

$$\dot{\boldsymbol{\omega}}_f = -k_p\xi_{bt} - k_d\omega_f + \boldsymbol{J}^{-1}(\boldsymbol{W}_f\boldsymbol{\theta} + \boldsymbol{u}_f) + \boldsymbol{\gamma} \tag{6}$$

where $\boldsymbol{\gamma} = \boldsymbol{\gamma}(0)\mathrm{e}^{-\kappa t}$ is an exponentially vanishing term, and its initial value related to system states: $\boldsymbol{\gamma}(0) = \boldsymbol{\omega}_f(0) + k_p\boldsymbol{\xi}_{bt}(0) + k_d\boldsymbol{\omega}_f(0) - \boldsymbol{J}^{-1}(\boldsymbol{W}_f(0)\boldsymbol{\theta} + \boldsymbol{u}_f(0)) = \boldsymbol{\omega}_{bt}^b(0) + k_p\boldsymbol{\xi}_{bt}(0) - \boldsymbol{J}^{-1}\boldsymbol{u}_f(0)$, The main reason for us to build this filtered structure is that now the filtered angular velocity $\boldsymbol{\omega}_f$ is attainable, and the consequence of this fact is discussed as follows.

1) Consider an auxiliary variable $\boldsymbol{\beta}$ defined by $\boldsymbol{\beta} = \dot{\boldsymbol{\omega}}_f - k_p\boldsymbol{\xi}_{bt} - k_d\boldsymbol{\omega}_f + \boldsymbol{\gamma}$. Then, if $\boldsymbol{o}_f$ is designed to follow the form $\boldsymbol{u}_f = \boldsymbol{W}_f\hat{\boldsymbol{\theta}}$, where $\hat{\boldsymbol{\theta}}$ denotes the adaptive estimate of $\boldsymbol{\theta}$, we have $\boldsymbol{u}_f = \mathbf{0}_3$. Accordingly, one can readily obtain that $\boldsymbol{\gamma}(t)$ and $\boldsymbol{\beta}(t)$ are also an attainable variable for all $t \geq 0$, and $\boldsymbol{\beta} = \boldsymbol{J}^{-1}\boldsymbol{W}_f\tilde{\boldsymbol{\theta}}$, where $\tilde{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}} - \boldsymbol{\theta}$. It follows that $\boldsymbol{\beta}$ contains the information concerning the parameter estimation errors.

2) Recalling Eq. (6) and once again employing the affine representation property of attitude dynamics, we have $\boldsymbol{W}_a\boldsymbol{\theta}=\boldsymbol{u}_f$, where the new regressor matrix $\boldsymbol{W}_a$ satisfies $\boldsymbol{W}_a\boldsymbol{\theta}=\boldsymbol{J}\dot{\boldsymbol{\omega}}_f+k_p\boldsymbol{J}\boldsymbol{\xi}_{bt}+k_d\boldsymbol{J}\boldsymbol{\omega}_f-\boldsymbol{J}\boldsymbol{\gamma}-\boldsymbol{W}_f\boldsymbol{\theta}$ and thus $\boldsymbol{W}_a(t)$ is also attainable for all $t\geq 0$. This result indicates that, by introducing filtered states and regressor matrices, the information of the unknown parameter vector $\boldsymbol{\theta}$ (coupled with $\boldsymbol{W}_a$) can be acquired through the auxiliary control input $\boldsymbol{u}_f$.

After all these preliminaries, the main result of this paper, a composite adaptive controller for attitude tracking control of rigid bodies, is summarized in the following theorem.

Theorem 1: Consider the attitude tracking error dynamical model in Eqs. (1) and (2), and further consider the filtered states, regressor matrices, and dynamics in Eqs. (4), (5), and (6). The control torque signal and adaptive law are defined as follows,

$$\boldsymbol{u}_c(t)=\dot{\boldsymbol{u}}_f(t)+\kappa\boldsymbol{u}_f(t),\quad \boldsymbol{u}_f(t)=\boldsymbol{W}(t)\hat{\boldsymbol{\theta}}(t) \tag{7}$$

$$\dot{\hat{\boldsymbol{\theta}}}(t)=-(\frac{k_a}{k_p+k_d}+k_a\mu)\boldsymbol{W}_f^{\mathrm{T}}(t)\boldsymbol{\beta}(t)-k_ak_l\boldsymbol{\Omega}_a(t) \tag{8}$$

where k_a, k_l and μ are positive constants, the definition of $\boldsymbol{\Omega}_a(t)$ is

$$\boldsymbol{\Omega}_a(t)=\begin{cases}\boldsymbol{C}(t)[\boldsymbol{A}(t)\hat{\boldsymbol{\theta}}(t)-\boldsymbol{B}(t)] & \text{if } \forall t_a\in[0,t], \mathrm{rank}(\boldsymbol{A}(t_a))<6\\ \boldsymbol{A}^{-1}(t_a)[\boldsymbol{A}(t_a)\hat{\boldsymbol{\theta}}(t)-\boldsymbol{B}(t_a)] & t_a=\min\{\arg_{\tau\in[0,t]}[\mathrm{rank}(\boldsymbol{A}(\tau))=6]\}\end{cases} \tag{9}$$

with $\boldsymbol{C}(t)=\boldsymbol{A}^{\mathrm{T}}(t)[\boldsymbol{A}^{\mathrm{T}}(t)\boldsymbol{A}(t)+a\mathbf{I}_{6\times 6}]^{-1}$, and the matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ are defined by

$$\dot{\boldsymbol{A}}(t)=-\sigma\boldsymbol{A}(t)+\boldsymbol{W}_a^{\mathrm{T}}(t)\boldsymbol{W}_a(t),\quad \boldsymbol{A}(0)=\mathbf{0}_{6\times 6} \tag{10}$$

$$\dot{\boldsymbol{B}}(t)=-\sigma\boldsymbol{B}(t)+\boldsymbol{W}_a^{\mathrm{T}}(t)\boldsymbol{u}_f(t),\quad \boldsymbol{B}(0)=\mathbf{0}_{6\times 1} \tag{11}$$

wherein a and σ are positive constants. If $\boldsymbol{W}_a(t)$ further satisfies the FE condition as given in Definition 1, then, for arbitrary $\boldsymbol{q}_{bt}(0)\in\mathbb{H}$, $\boldsymbol{\omega}_{bt}(0)\in\mathbb{R}^3$ and $\hat{\boldsymbol{\theta}}(0)\in\mathbb{R}^6$, it follows that $\lim_{t\to\infty}\{\boldsymbol{\xi}_{bt}(t),\boldsymbol{\omega}_{bt}(t),\tilde{\boldsymbol{\theta}}(t)\}=\mathbf{0}_3$.

Proof: To analyze the closed-loop stability, employ the following storage function,

$$V=(\boldsymbol{q}_{bt}-\boldsymbol{q}_I)^{\mathrm{T}}(\boldsymbol{q}_{bt}-\boldsymbol{q}_I)+\frac{1}{2}\boldsymbol{\omega}_f^{\mathrm{T}}\boldsymbol{\omega}_f+\frac{\varrho}{2}\boldsymbol{\gamma}^{\mathrm{T}}\boldsymbol{\gamma}+\frac{\sigma}{2k_a}\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\tilde{\boldsymbol{\theta}} \tag{12}$$

where $\boldsymbol{q}_I=[1,0,0,0]^{\mathrm{T}}$ is the identity quaternion, ρ and σ are positive constants which are introduced just for the sake of analysis, they satisfy $\rho=1/(\kappa k_p)+1/(\kappa k_d)$ and $\sigma=1/J_{\mathrm{m}}$, wherein J_m denotes the minimum eigenvalue of $\boldsymbol{J}$. Then, substituting Eqs. (2), (7) and (8) into the time derivative of V, and employing the Cauchy-Schwarz inequality, one can obtain

$$\dot{V}\leq-\frac{k_p}{4}\boldsymbol{\xi}_{bt}^{\mathrm{T}}\boldsymbol{\xi}_{bt}-\frac{k_d}{4}\boldsymbol{\omega}_f^{\mathrm{T}}\boldsymbol{\omega}_f-\frac{\kappa\rho}{2}\boldsymbol{\gamma}^{\mathrm{T}}\boldsymbol{\gamma}-\sigma\mu\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{W}_f^{\mathrm{T}}\boldsymbol{J}^{-1}\boldsymbol{W}_f\tilde{\boldsymbol{\theta}}-k_l\sigma\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\Omega}_a \tag{13}$$

When $\boldsymbol{W}_a$ satisfies the FE condition, by the definition of FE, there exist finite constants $t^*\geq 0$, $T>0$ and $c>0$, such that

$$\int_{t^*}^{t^*+T}\boldsymbol{W}_a^{\mathrm{T}}(\tau)\boldsymbol{W}_a(\tau)\geq c\mathbf{I}_{m\times m} \tag{14}$$

Then, from Eq. (10), one has

$$\boldsymbol{A}(t^*+T)\geq \mathrm{e}^{-\sigma(t^*+T)}\mathrm{e}^{\sigma t^*}\int_{t^*}^{t^*+T}\boldsymbol{W}_a^{\mathrm{T}}(\tau)\boldsymbol{W}_a(\tau)\mathrm{d}\tau\geq c\mathrm{e}^{-\sigma T}\mathbf{I}_{6\times 6} \tag{15}$$

This result indicates that, if $\boldsymbol{W}_a$ satisfies the FE condition, there always exists a finite time $t_a = t^* + T$, such that $\forall t \geq t_a$, $\boldsymbol{A}(t)$ is full-rank. Accordingly, by Eq. (13), for all $t \geq t_a$, the storage function V satisfies

$$\dot{V}(t) \leq -\frac{k_p}{4}\| \boldsymbol{\xi}_{bt}(t)\|^2 - \frac{k_d}{4}\| \boldsymbol{\omega}_f(t)\|^2 - \frac{\kappa\rho}{2}\| \boldsymbol{\gamma}(t)\|^2 - \sigma\mu J_m \| \boldsymbol{J}^{-1}\boldsymbol{W}_f(t)\tilde{\boldsymbol{\theta}}(t)\|^2 - k_l\sigma\| \tilde{\theta}(t)\|^2$$

Subsequently, by further employing Barbalat's lemma, one can finally ensure that $\lim_{t\to\infty}\{\boldsymbol{\xi}_{bt}(t), \boldsymbol{\omega}_{bt}(t), \tilde{\theta}(t)\} = \mathbf{0}_3$. The proof is complete.

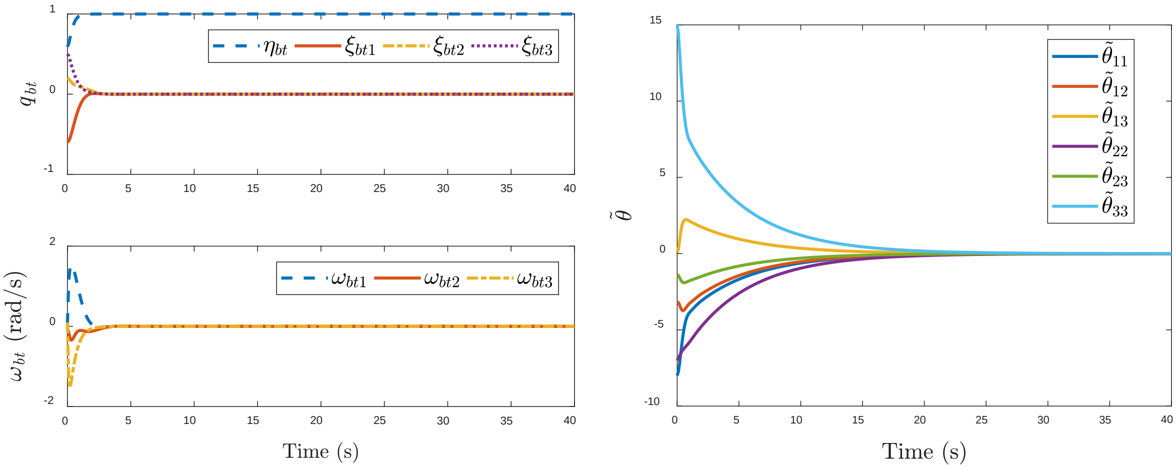
Numerical Simulations

To evaluate the performance of the proposed method regarding parameter convergence, an attitude stabilization problem (i.e. tracking a non-PE zero reference trajectory: $\boldsymbol{\omega}_{ti}^t = [0,0,0]^{\mathrm{T}}$) is employed. The parameter vector of the rigid body is $\boldsymbol{\theta}$=[20,1.2,0.9,17,1.4,15]$^{\mathrm{T}}$ kg·m^2, initial states are set to be $\boldsymbol{q}_{bt} = [0.5916, -0.6, 0.2, 0.5]^{\mathrm{T}}$, $\boldsymbol{\omega}_{bt}^b = [0.05, -0.05, 0.1]^{\mathrm{T}}$ rad/s, and $\hat{\boldsymbol{\theta}}$=[12,-2,1,10,0,30]$^{\mathrm{T}}$ kg·m^2. Control gains are k_p=2, k_d=5, μ=10, k_a=1, k_l=1, σ=0.01, a=0.05, and the simulation step is set to be 0.1s.

Time histories of tracking errors and also parameter estimation errors are illustrated in Fig. 1. A remarkable feature observed in this figure is that the parameter estimation error vector $\hat{\boldsymbol{\theta}}$ asymptotically converges to zero even when tracking errors are (nearly) eliminated ($\boldsymbol{q}_{bt} \to \boldsymbol{q}_I$, $\boldsymbol{\omega}_{bt}^b \to \mathbf{0}_3$), which is impossible to be achieved by employing conventional adaptive control methods in stabilization problems. To further analyze the convergence process of the inertia estimates, the trajectories of main elements (diagonal entries) of $\hat{\boldsymbol{\theta}}$=$\hat{\theta}_{11}, \hat{\theta}_{22}, \hat{\theta}_{33}$, are illustrated in Figs. 3 and 4, and the update directions induced by current data (the I\&I-based part in adaptive law) and historical data (the learning part in adaptive law) of every step are indicated by blue arrows and red arrows, respectively. It is evident that, at the beginning (the transient phase), convergence trajectories of estimates are driven by both current data and historical data, then the influence of current data decreases with the decaying of state errors (steady-state phase), while under the effect of historical data, all parameters are still able to converge to their corresponding true values even when the reference signal is not PE.

Conclusion

A novel adaptive controller for attitude tracking control problems of rigid bodies is proposed in this paper. The most important feature of this new adaptive scheme is the guarantee of precise convergence of not only state tracking errors but also parameter estimation errors, subject to the satisfaction of a finite excitation condition. The overall implication of this result is improved closed-loop performance which comes at the added cost in terms of buffer memory that is required for retaining measurements of past state values (historical data). Numerical simulation results illustrate the various features of the proposed method. Further work in this direction would consider robustness modifications to account of imperfect measurements and possible presence of unmodeled dynamics.



(a) system states (b) parameter estimation errors

Fig. 1 Simulation results of system states and parameter estimation errors

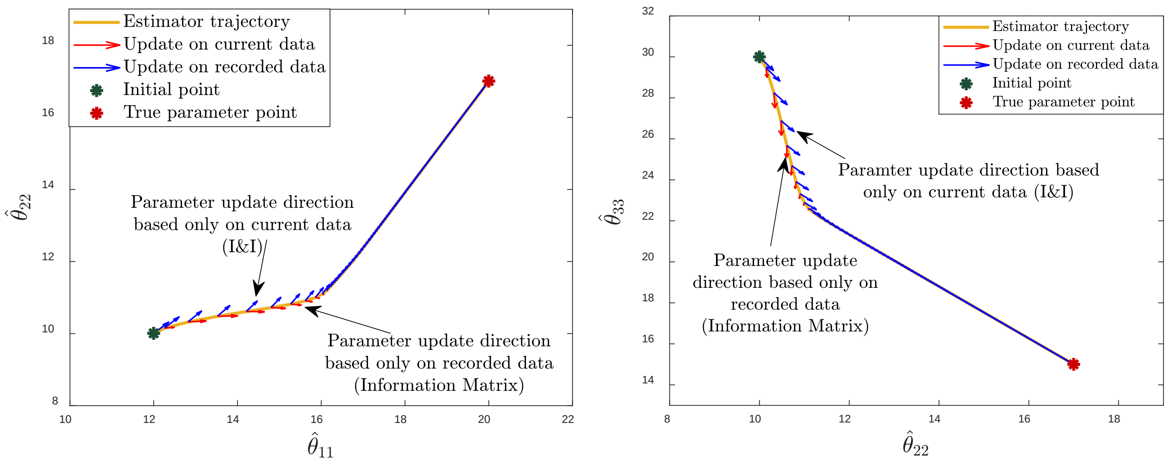


Fig. 2 Convergence trajectories of $\hat{\theta}_{11}, \hat{\theta}_{22}, \hat{\theta}_{33}$

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