18th Australian International Aerospace Congress
27th International Symposium on Space Flight Dynamics

Please select category below:
Normal Paper ☒
Student Paper ☐
Young Engineer Paper ☐

Utilising the 'Chaotic' Tumbling of CubeSats

Graham E. Dorrington[1] and Pavel M. Trivailo[1]

[1]*School of Engineering, Royal Melbourne Institute of Technology, PO Box 71, Bundoora, Victoria, 3083, Australia*

Abstract

The tumbling dynamics of a solid-cube body representative of a passive 1-CU CubeSat are simulated. It is assumed that the CubeSat user is interested in maximising a directional attitude pointing preference, e.g., maximising the transmission rate from a patch antenna, or some instrument mounted on one body face, without the use of any active attitude control system. The inertia values of the body are assumed to be different in the three body axes, such that there is a major inertia axis, an intermediate inertia axis and a minor inertia axis. Simulation confirms the body undergoes Dzhanibekov flipping when it is rotating about the intermediate axis. An efficiency measure is defined that quantifies the percentage of time a chosen body face is pointing in the desired direction. It is found that use of the face perpendicular to the intermediate axis offers the highest efficiency and moreover the efficiency never falls to zero, whereas use of a face perpendicular to the major axis results in the possibility of zero efficiency. This result appears to be a general one and appears to apply for all initial conditions, provided Dzhanibekov flipping occurs.

Keywords: inertia, attitude, control, transmission, nanosat.

Introduction

CubeSats and similar nanosats are often constrained by quite limited mass and cost budgets. In some cases the use of an active attitude control system (e.g., magnetorquers) may not be viable. After release from a launch pod, such a passive CubeSat will often be left tumbling in a seemingly chaotic manner with rotation rates of ~1 rad/s. The designer may also be limited in terms of subsystem options. For example, if a single patch antenna is employed on just one face of a 1U CubeSat face and the other faces are dedicated to other instruments or subsystems, then communications will be interrupted when the antenna is pointing in the wrong direction. Indeed, this could be a potential mission hazard: the CubeSat could remain in a stable coning rotation with an undesirable rotation axis direction such that the antenna is pointing away from the intended origin point (e.g., a receiving station) indefinitely, see Fig, 1.

To overcome this potential problem, we propose the "Dzhanibekov Effect" [e.g., 1, 2] may be usefully employed [3]. We assume the CubeSat has three dissimilar inertia values in the three body axes: the largest value for the "major" axis, the lowest value for the "minor" axis, and an intermediate value for the "intermediate" axis. If the body is initially rotating about the intermediate axis, then it will experience periodic flipping whereby the faces perpendicular to this axis effectively swap sides [1, 3].

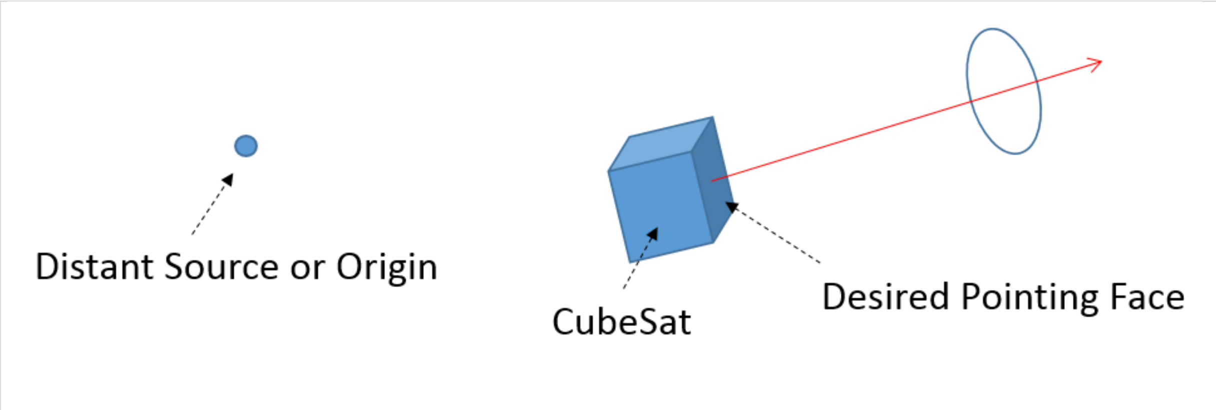


Fig. 1: Problem of stable rotation about axis (red arrow) leading to desired pointing face not seeing a distant origin or source.

We define the "pointing efficiency" as the integral over one complete rotation of cos(β) where β is the angle between a chosen body axis and a vector from the body center to some designated distant origin - where the user wishes to point (see Fig.2). We prescribe different body positions using two angles in the absolute frame and prescribe different initial rotation states. We find that the pointing efficiency may drop to zero when face perpendicular to the major axis is chosen, but never drops to zero when the intermediate axis is chosen and it reaches comparatively high values in the latter case. This appears to be a general finding. In other words, for the patch antenna example briefly described above, it is beneficial to mount the antenna on one of the two faces that are perpendicular to the intermediate axis.

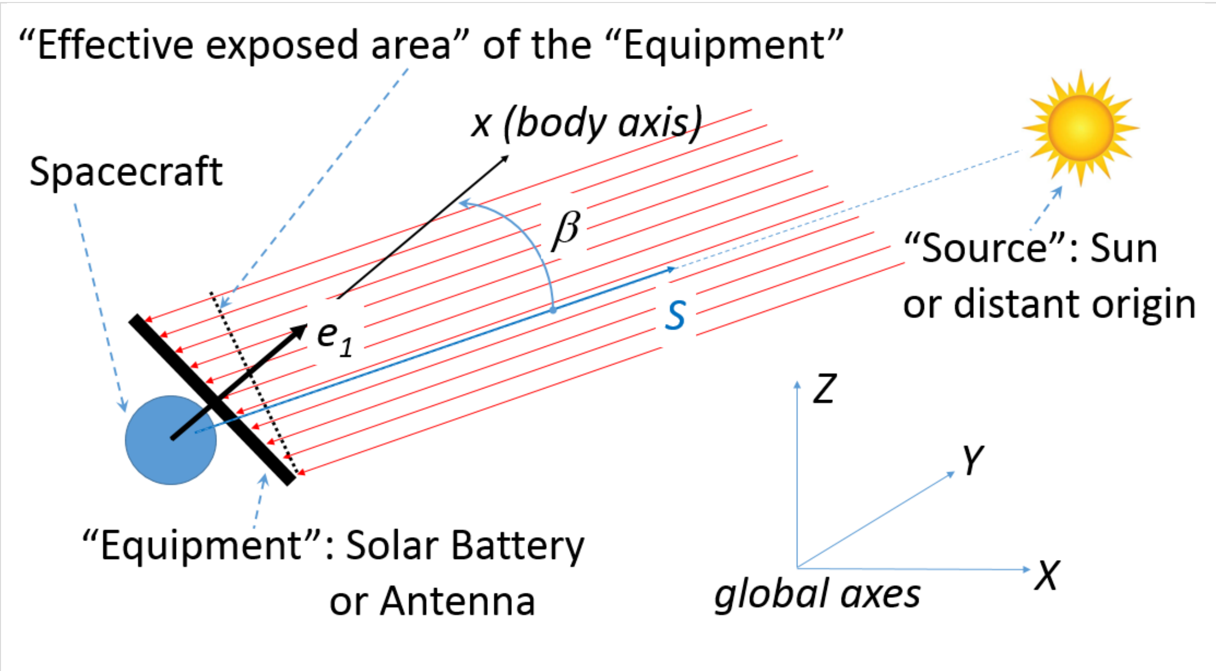


Fig. 2: Introduction of the "Effective exposed area" and notations: S-direction to the "source", e_1-unit ort vector along "x" body axis, β-effective angle.

Description of Simulation

We modelled the CubeSat rotation rate ω, with constant moments of inertia, I, corresponding to the body axes, x, y and z using the well-known Euler equations [4] for a rigid body,

$$\begin{cases} I_{xx}\,\dot{\omega}_x = (I_{yy} - I_{zz})\,\omega_y\,\omega_z \\ I_{yy}\,\dot{\omega}_y = (I_{zz} - I_{xx})\,\omega_z\,\omega_x \\ I_{zz}\,\dot{\omega}_z = (I_{xx} - I_{yy})\,\omega_x\,\omega_y \end{cases} \tag{1}$$

We selected different arbitrary parameters and initial conditions, e.g. for the case presented in the following section we chose: I_{xx} = 2 kg m^2 ; I_{yy} = 4 kg m^2; I_{zz} =3 kg m^2. This corresponds to minor, major and intermediate inertia values, respectively. We chose the following initial conditions: ω_x = 0.01 rad/s, ω_y = 0.01 rad/s, ω_z = 1 rad/s. Before the simulation was run, we aligned the body axes of the spacecraft as follows: "*x*"-body axis is aligned with "*X*" global coordinate system axis; "*y*" body axis is aligned with "*Y*" global coordinate system axis; "*z*" body axis is aligned with "-*Z*" global coordinate system axis. As the predominant rotation is provided along the *z* axis, being the so called "intermediate" axis of inertia, the resulting motion was expected and found to result in Dzhanibekov flipping.

We introduced an "effective" angle β – the angle between the direction to the "source" *S* and selected body axis, *x*, *y* or *z*. We assume that the efficiency of the chosen set-up is proportional to the cosine of the effective angle β, such that when cos(β) is positive and closer to 1, the instantaneous efficiency is considered to be better, since the "effective exposed area" of the antenna, or solar panel, or other "equipment" mounted on one CubeSat face is larger. When cos(β)=1, the efficiency is at its maximum. When cos(β) is negative, we assume zero efficiency. In addition to the time history of the cos(β), we can also plot the time integral of the cos(β), which measures the total accumulated efficiency of the system.

We considered the following cases:

i) *S*=[0 -1 0], (i.e. *S* is pointing in the –*Y* direction);
ii) *S*=[0 1 0], (i.e. *S* is pointing in the +*Y* direction);
iii) *S*=[-1 0 0], (i.e. *S* is pointing in the –*X* direction).

Results

For the purpose of presenting the position directions of the unit vectors in the body axes system, we consecutively plot the trajectories of their intersections with a sphere centered at the centre of the mass of the 1U CubeSat. The simulation results for the first 200 s are presented separately for "*x*", "*y*", and "*z*" axes in Figure 3 in three different rows, where each row has the same results, but presented from two contrasting view-points. Figure 3 shows, that for the considered study case, the face perpendicular to either the "*x*", "*y*" or "*z*" axes, has very different exposure to desired pointing direction. For example, as seen from Figures 3 (c) and (d), a face with equipment aligned pointing towards the +*Y* global axis will not have any exposure with the desired pointing direction. Also, the face aligned with "*x*" body axis will have relatively short-limited exposure.

Figure 4 and 5 show the time histories. Fig. 4c shows that for the simulated case for the system with *S* vector (pointing towards the "source") coinciding with e_2, the efficiency of the system is zero. In other words, in this case placing (for example) a patch antenna on the face perpendicular to the major axis would result in no communication with the source, simply because that face remains permanently facing away from that source.

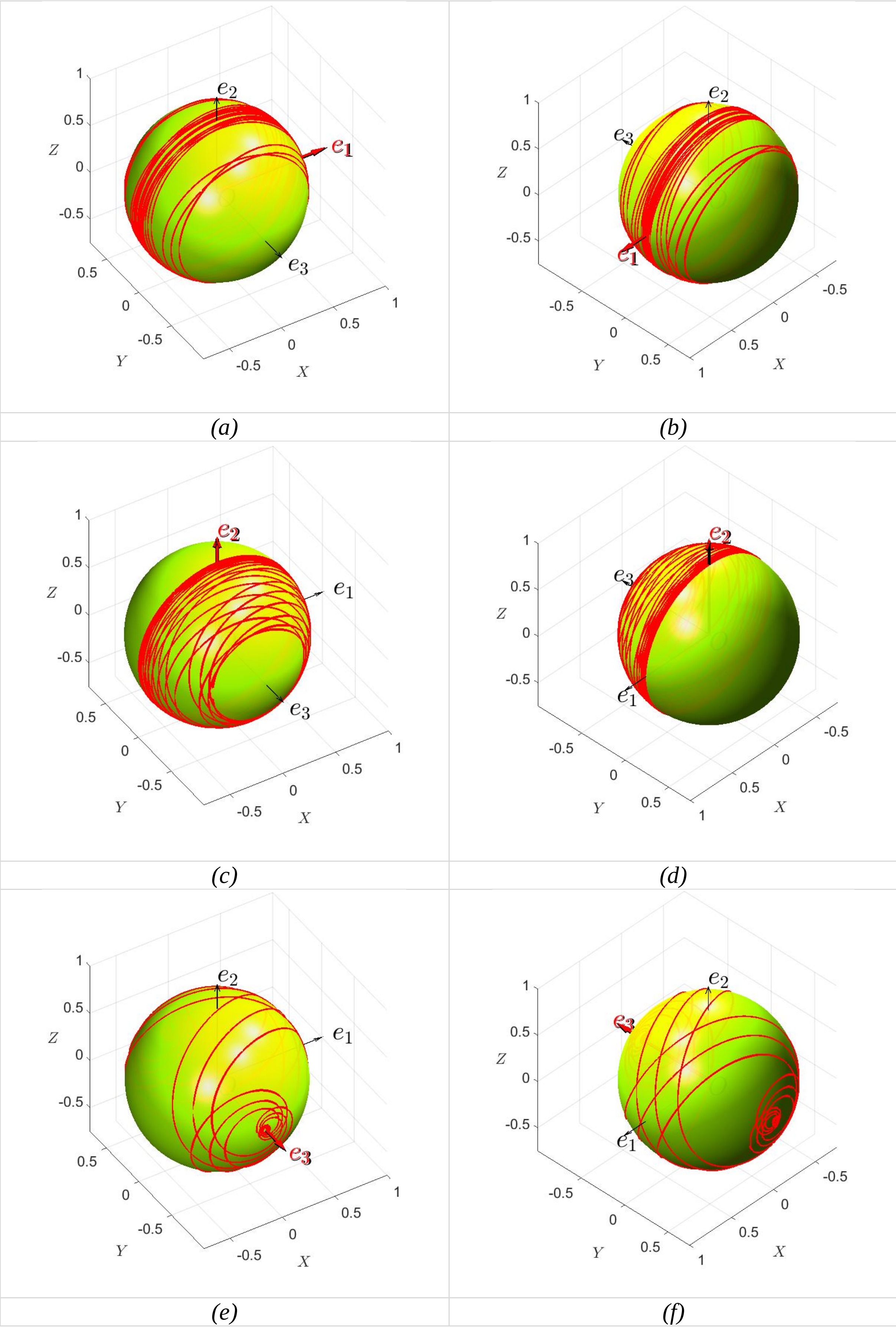
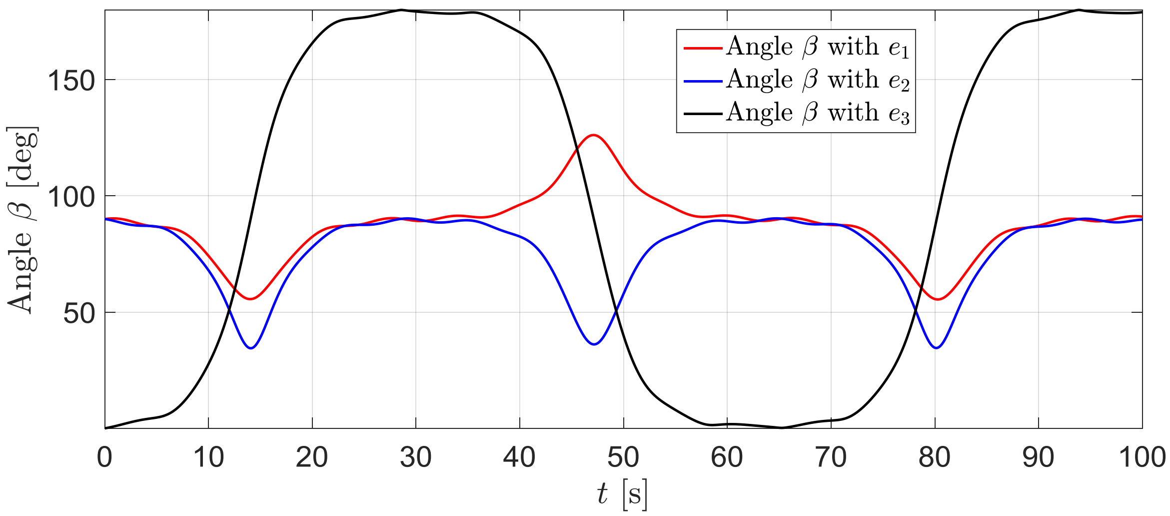
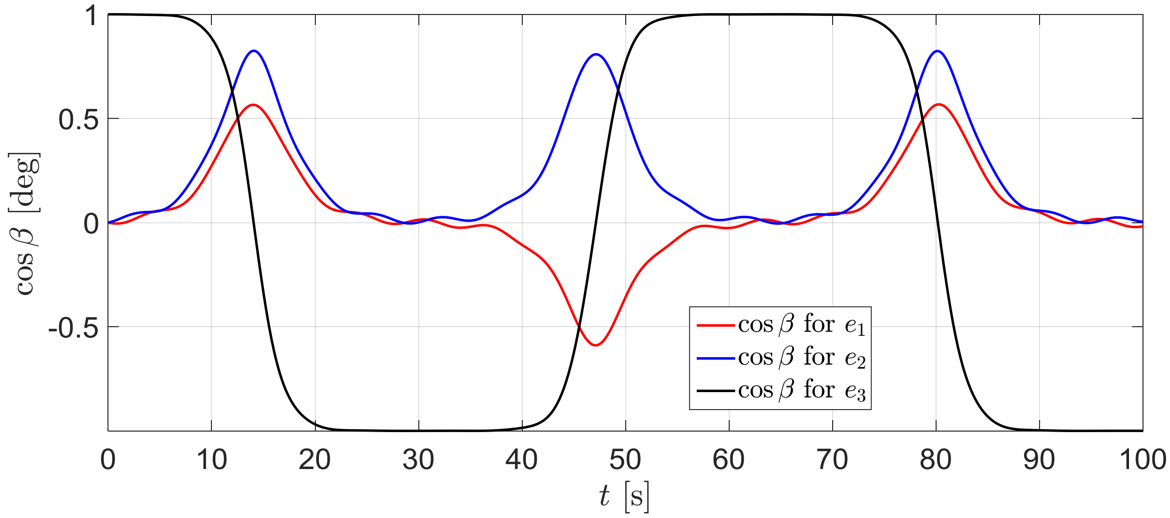


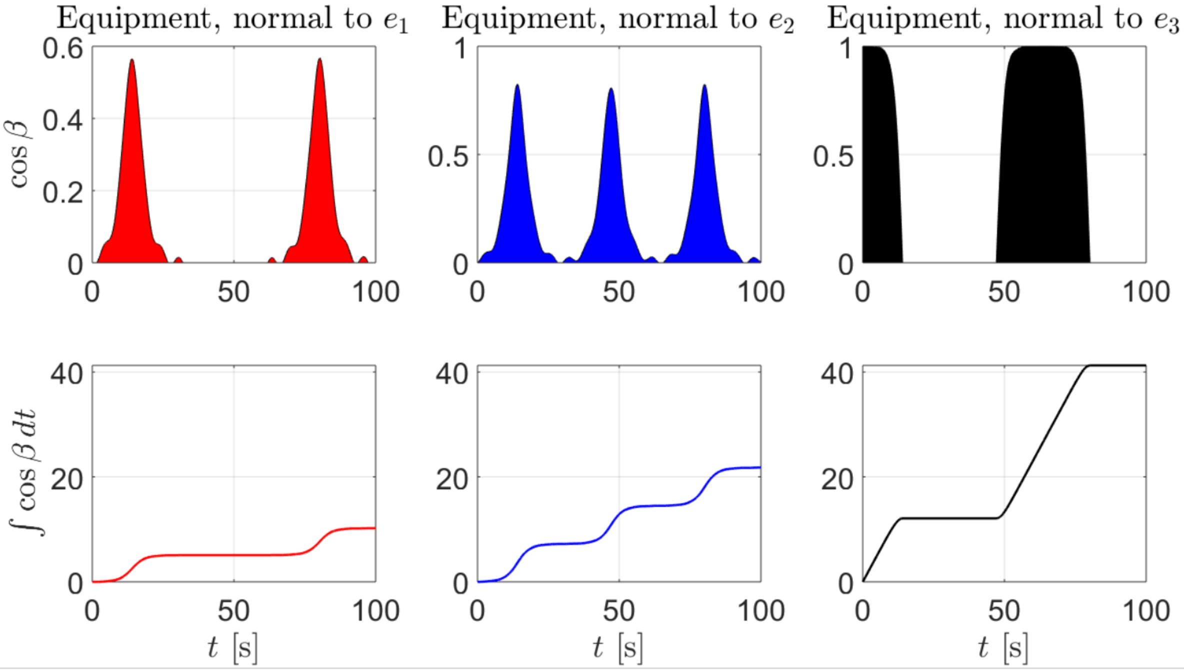
Fig. 3: Intersection of the unit vectors of the body axes system with a body sphere: (a, b) "x" (minor) body axis; (c, d) "y"(major) body axis; (e, f) "z" body axis (intermediate).



(a)

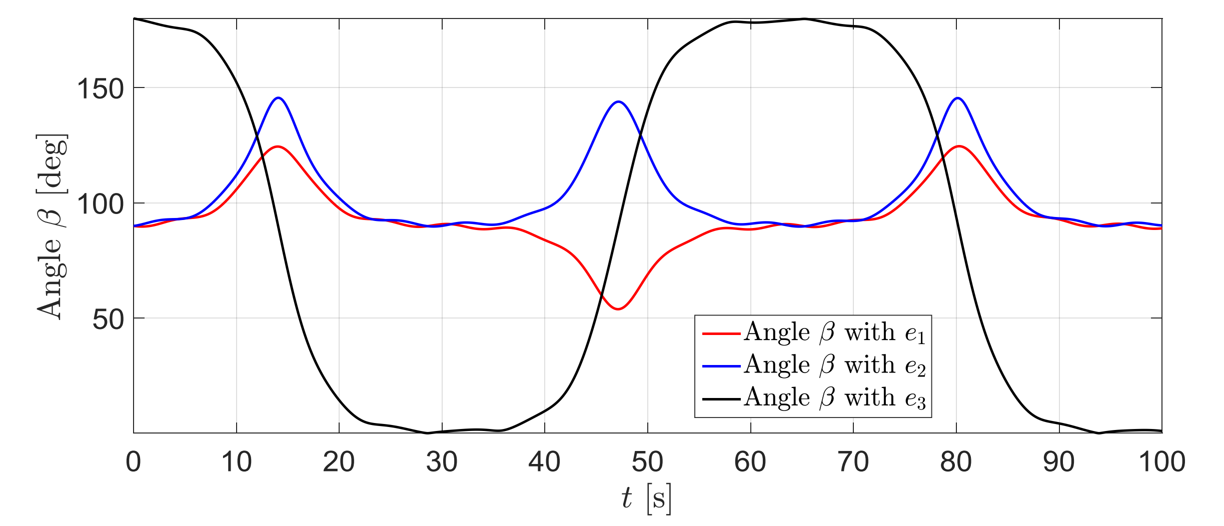


(b)

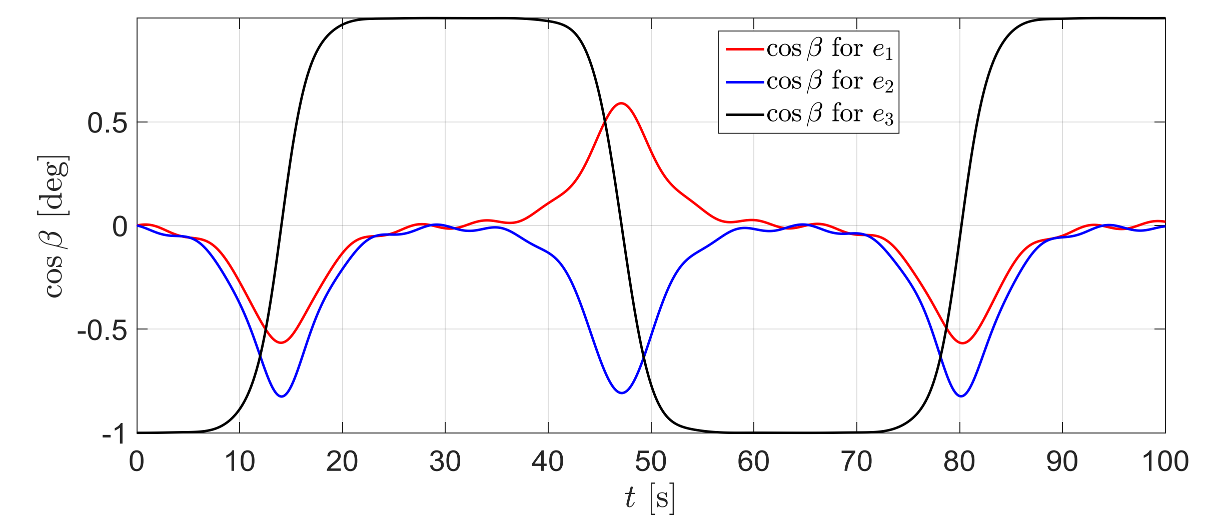


(c)

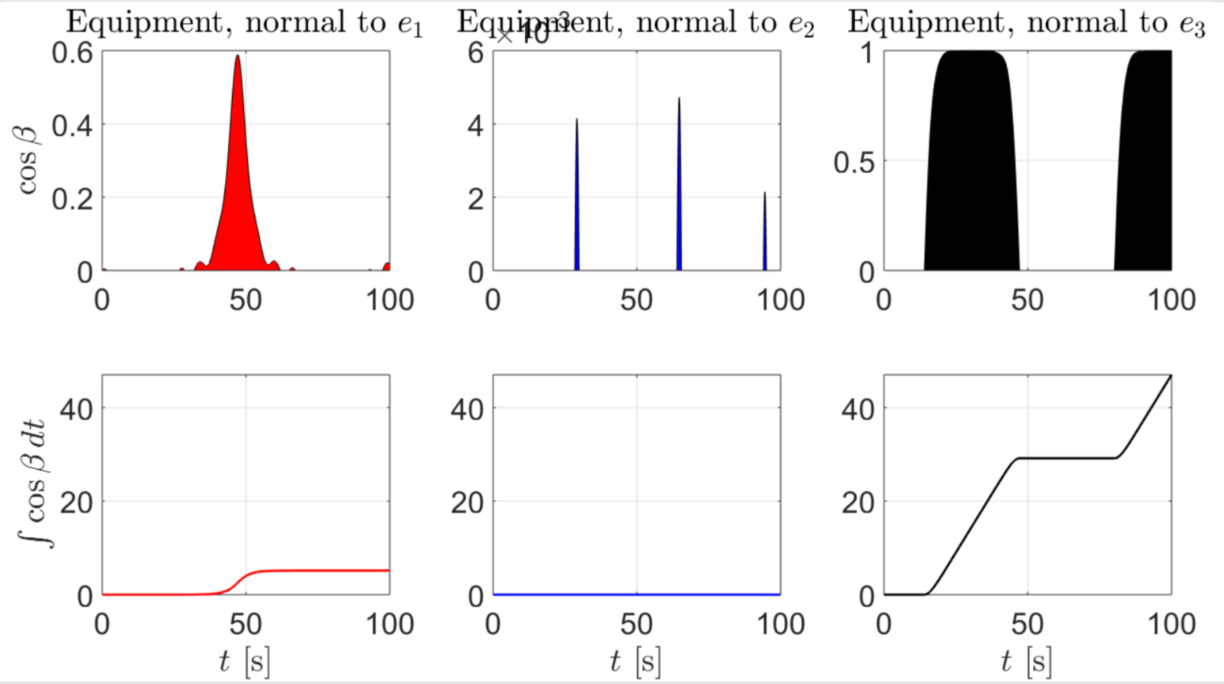
Fig. 4: S-direction to the "source" is given with the [0 -1 0] vector.



(a)



(b)



(c)

Fig. 5: S-direction to the "source" is given with the [0 1 0] vector.

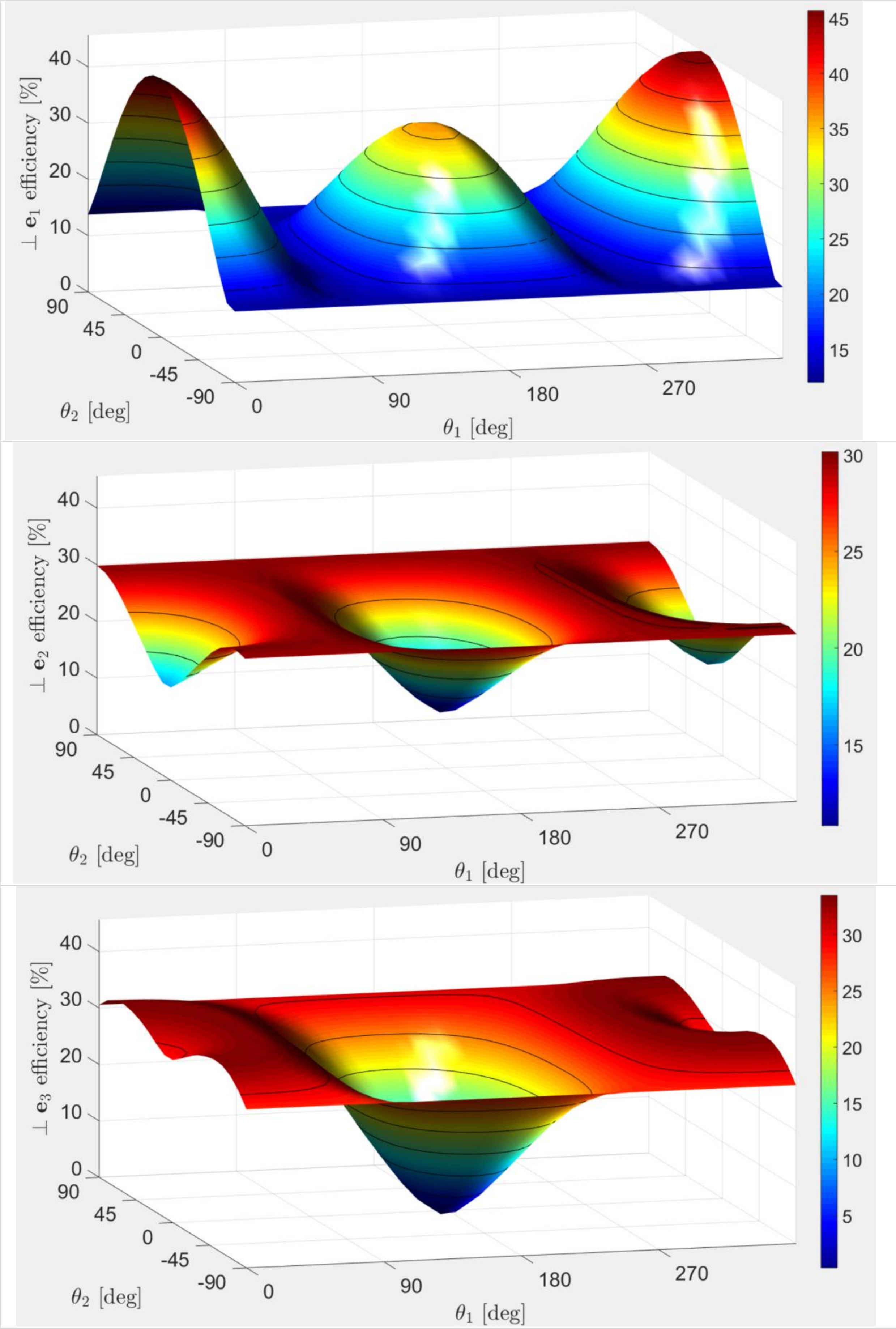


Fig. 6: Plots of pointing efficiency with chosen faces:
intermediate (top), minor (middle) and major (bottom).

Figure 6 shows some efficiency plots where $\theta_{1,2}$ are the angles of vector from the source to the centre of the spacecraft. For this example, the maximum and minimum values for integrated efficiency, for the faces e_1, e_2 and e_3, corresponding to faces perpendicular to the intermediate, minor and major axis, respectively, are given in Table 1.

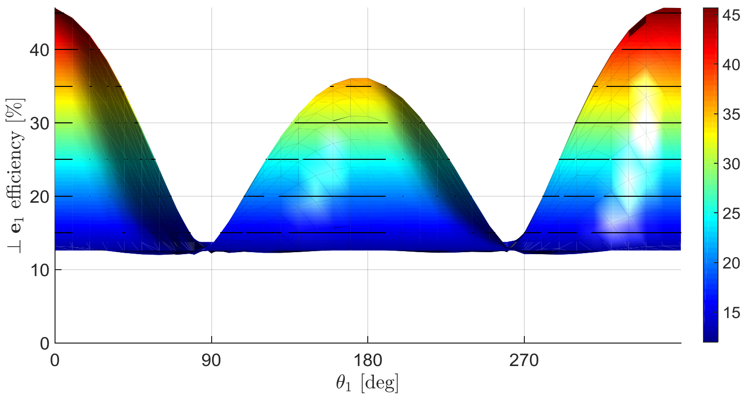
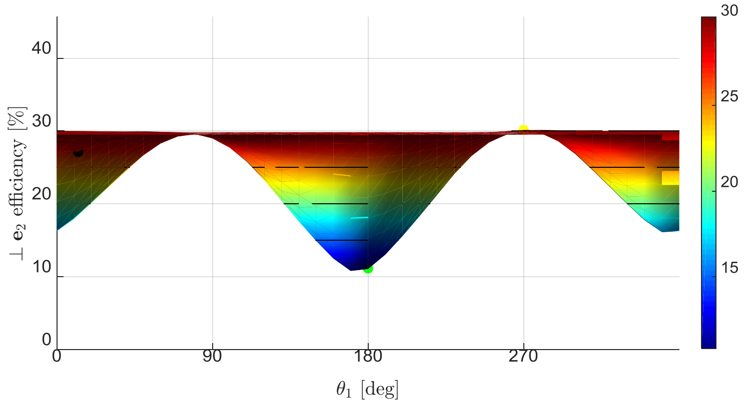
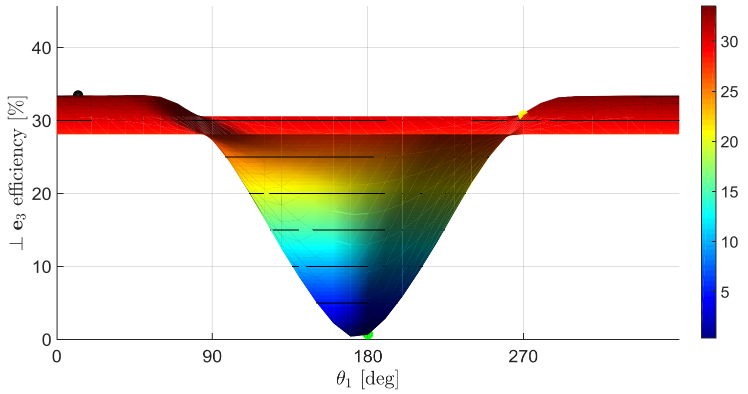
Face	Maximum %	Minimum %	Graphical Illustration, using side view
e_1	45.7	12.0	
e_2	30.2	10.8	
e_3	33	0.3	

Table 1: Maximum and Minimum Integrated Pointed Efficiency Values for Different Selected Cube Faces.

After running many similar such test cases, we believe that this example represents a general result. In other words, mounting the relevant equipment on the face perpendicular to intermediate axis can offer relatively high maximum pointing efficiency and also avoid the hazard of unacceptably low pointing efficiency. We also modified our efficiency definition such that the efficiency was unity when pointing within a tolerance angle of 40 degrees and zero when outside this cone and obtained a similar result.

Concluding Remarks

Our results demonstrate that there is a benefit in mounting a directional-pointing subsystem (such as a patch antenna, or a solar panel, or any instrument that needs to be directed to a distant origin or source) on 1U CubeSat faces perpendicular to the axis with an intermediate inertia value. This result appears to be a general one for a broad range of initial conditions and pointing directions, provided that the CubeSat is released from a pod in a manner that results in rotation about the intermediate axis, i.e., in manner that results in Dzhanibekov flipping. Therefore, we believe we have demonstrated the Dzanibekov Effect can be utilised beneficially in some CubeSat operations.

References

1. Murakami, H., Rios, O., Impelluso, T. J.: A Theoretical and Numerical Study of the Dzhanibekov and Tennis Racket Phenomena, *Journal Applied Mechanics*, 83 (2016), 111006 (10 pages). Paper No: JAM-16-1017, DOI:10.1115/1.4034318.

2. Trivailo, P.M., Kojima, H., Utilisation of the "Dzhanibekov's Effect" for the Possible Future Space Missions, Paper ISTS-2017-d-047/ISSFD-2017-047, *Proceedings of the 31st International Symposium on Space Technology and Science (ISTS), 26th International Symposium on Space Flight Dynamics (ISSFD) & 8th NanoSatellite Symposium (NSAT)*, Matsuyama, Japan, 3-9 June 2017. – 10pp.

3. Trivailo, P.M., Kojima, H., Discovering Method of Control of the "Dzhanibekov's Effect" and Proposing its Applications for the Possible Future Space Missions. - *Transaction of JSASS (The Japan Society for Aeronautical and Space Sciences), Aerospace Technology Japan*, 2019, Vol. 17, No. 1, pp. 72-81. DOI: 10.2322/tastj.17.72.

4. Wie, B., "*Space Vehicle Dynamics and Control*", Second Edition, AIAA Education Series, 2008. – 966 pp, DOI:10.2514/4.860119.