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Spatial Formation of High Inclined Orbits with Use of Gravity Assists

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Abstract

Low-cost interplanetary tours with the high inclined orbit's formation in the Solar system using gravity assists maneuvers (GAMs) near planets (Earth and Venus) and with the accurate ephemeris involving are considered. Limited dynamic opportunities of their use require multiple passes near them. Topicality of the regular creation of optimum scenarios – sequences of cranking passing of celestial bodies and the solution of conditions of their execution is obvious. This work is devoted to the description of required features of trajectory's beams for the creation of such chains. Previously a comparative analysis of various modern astrodynamics studies of the 3D implementation of GAMs taking into account accurate ephemerides was performed. Improved analytical formulas for the change of inclination of an SC orbit as a result of 3D GAMs were obtained and realistic results of the computation of parameters of the SC orbit inclination changes at the planets of the Solar system and their moons were presented. In this paper, we describe algorithms for designing multi-pass chains of GAMs that result in an energy-efficient increase of the inclination of the SC orbit to the ecliptic plane. Simultaneously, a generalization of the analytical Eqns for the general case of elliptic orbits of the SC and the partner planet for the GAM is obtained. Applications of its using for the study of concrete options of mission "InterhelioZond" are given.

Keywords: gravity assist maneuver, V-infinity globe, maximum inclination pole, resonant asymptotic velocity, invariant lines of resonances, "InterhelioZond" mission.

Introduction

Presently, a number of space missions aimed at the study of the interior heliosphere using the non-ecliptic positions of a spacecraft (SC) are under development (Solar Orbiter by ESA, Russian InterhelioZond etc.). Such projects require the SC orbit to have a high inclination relative the ecliptic plane. In astrodynamics, the maneuvers that actively change the orbit inclination are especially energy consuming. Therefore, to save energy, it is reasonable to use natural energy sources in the form of gravity assist maneuvers (GAMs). In connection with such space missions, researchers at NASA, ESA, and JAXA, and Russian researchers are intensively analysing practical schemes for the 3D implementation of GAMs.

Low-cost interplanetary tours with the high inclined orbit's formation in the Solar system using gravity assists maneuvers (GAMs) near planets (Earth and Venus) and with the accurate ephemeris involving are considered. Limited dynamic opportunities of their use require multiple passes near them. Topicality of the regular creation of optimum scenarios – sequences of cranking passing of celestial bodies and the solution of conditions of their execution is obvious. This work is devoted to the description of required features of trajectory's beams for the creation of such chains.

In [1,2,3] a comparative analysis of various modern astrodynamics studies [4,5,6,7,8] of the 3D implementation of GAMs taking into account accurate ephemerides was performed. Improved analytical Eqns for the change of inclination of an SC orbit as a result of 3D GAMs were obtained and realistic results of the computation of parameters of the SC orbit inclination changes at the planets of the Solar system and their moons were presented [1,2,3]. In this paper, we describe algorithms for designing multi-pass chains of GAMs that result in an energy-efficient increase of the inclination of the SC orbit to the ecliptic plane. Simultaneously, a generalization of the analytical Eqns derived in [1,2,3] for the general case of elliptic orbits of the SC and the partner planet for the GAM is obtained. Applications of its using for the study of concrete options of mission "InterhelioZond" are given.

As shown in the article the hopping between the main low-duration resonances when performing GAM near Venus (which require a change of 9.8-13.3 degrees) are effective up to the values $i_{\max} = 40°$ (strong GAMs, pumping or cranking). Otherwise (also in most cases of “lite” Mars and every time – for Mercury) it is possible only design a GAMs according mono-resonance sequence (weak, i.e. cranking GAMs).

Two fundamentally different classes of GAMs and their constraints

According [4] the calculation of the maximum rotation angle $\varphi = \varphi_{\max}$ of the spacecraft asymptotic velocity vector after a single GAM maneuver can be used:

$$\sin\frac{\varphi_{\max}}{2} = \frac{\mu}{\mu + (R_{pl} + h)V_\infty^2}, \tag{1}$$

where μ, R_{pl} is the planet’s gravitational parameter and it’s radius, h – the pericenter value.

The geometry of a GAM to a large degree is determined by whether the planet's velocity vector $\mathbf{V}_{pl}$ intersects the sphere of all possible virtual asymptotic velocities $\mathbf{V}_{\infty,\mathrm{out}}$ of the spacecraft relative to the planet constructed at the endpoint of the vector $\mathbf{V}_{pl}$ or the entire vector $\mathbf{V}_{pl}$ lies inside this sphere (see Fig. 1, the cases S_1, S_2 respectively). For the case S_1 the condition

$$v_\infty \equiv V_\infty / V_{pl} < 1, \tag{2}$$

where $V_\infty = |\mathbf{V}_{\infty,out}|$, must be satisfied. This sphere will be called the V_∞-sphere following the term V_∞-globe [5]. In the case of large (near hyperbolic) values of the spacecraft asymptotic velocity $V_\infty > V_{pl}$, arbitrary inclinations of the spacecraft orbit can be achieved using one flyby. Below, we assume that condition Eqn 2 is fulfilled.

Using Fig. 1 and [1,2], we define spherical coordinates — the radius V_∞ and the angles ρ and ψ. The coordinate ρ is the angle between the vector $\mathbf{V}_{\infty,\mathrm{out}}$ obtained after the gravitation maneuver and its projection on the plane of the orbit; ψ is the angle between this projection and the velocity vector of the planet $\mathbf{V}_{pl}$.

It is known [1,2,3] that the limitations of the change of the inclination i of the spacecraft orbit achieved by a GAM with the chosen planet (solo GAMs) can be interpreted as geometric and dynamic.

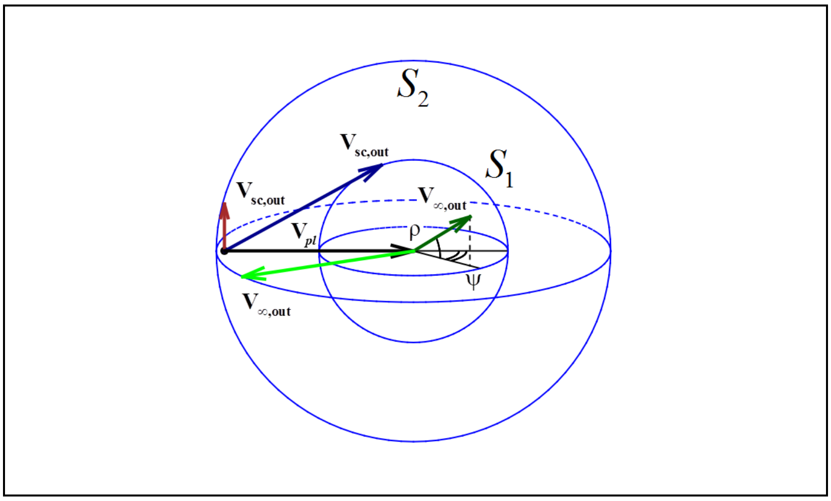

Fig. 1: Two different classes of GAMs: the case when the vector $\mathbf{V}_{pl}$ *intersects the* V_∞ *-sphere (* S_1 *) and when the vector* $\mathbf{V}_{pl}$ *lies inside this sphere*

The geometric limitations determine the maximum magnitude of i using any number of solo GAMs. For case Eqn 2, this magnitude is determined by the dimensionless asymptotic velocity of the spacecraft v_∞ [1,2,3,4,8]:

$$\sin i_{\max} = \frac{V_\infty}{V_{pl}} = v_\infty . \tag{3}$$

It is easy to see that Eqn 2 and Eqn 3 imply the constraint

$$i_{\max} < \frac{\pi}{2} . \tag{4}$$

The dynamic limitations of the change of the spacecraft orbit inclination are determined by the magnitude of the gravitational field of the planet and the minimum admissible flyby distance near it. The rotation angle φ of the vector of the spacecraft asymptotic velocity after a single GAM satisfies Eqn 1. The position of the inclination pole — the maximizer T_{Pole} of $i = i_{max}$ on the V_∞ -sphere is schematically shown in Fig. 2.

The meaning of the dynamic limitations for a GAM is that the endpoint of the output vector $\mathbf{V}_{\infty,out}$ after a GAM is within the spherical cap CK_φ. This domain is the intersection of the V_∞-sphere with the solid angle formed by the cone with the aperture 2φ and the axis coinciding with the input (before the GAM) asymptotic velocity vector $\mathbf{V}_{\infty,in}$ of the spacecraft. It is clear that the base of this domain CK_φ is the circle K_φ of radius $r_\infty = V_\infty \sin\varphi$ that is orthogonal to $\mathbf{V}_{\infty,in}$. The sequence of any solo GAMs aimed at increasing the spacecraft orbit inclination (they will be called *increasing* or *rising* chains) must approach the point T_{Pole} (Fig. 3) along any main resonant line (synchronisms between SC orbital period and GAM planetary orbital period are {3:4; 1:1; 4:3;…}) [1,2,3,9].

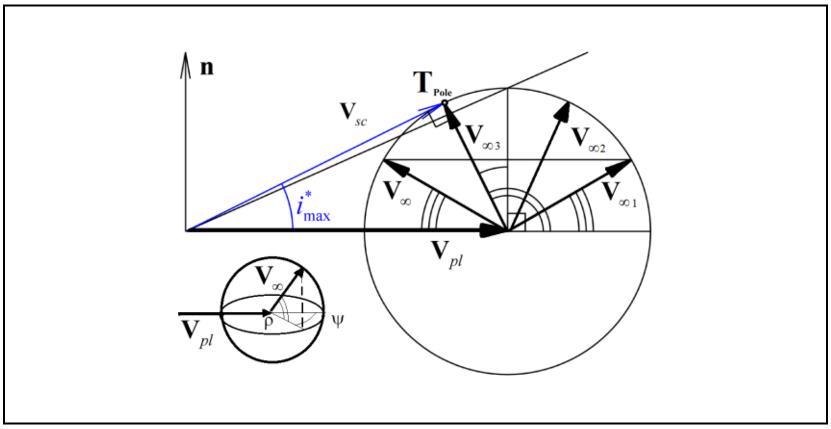

Fig. 2: Position of the inclination pole T_{Pole} *, the maximum* $i = i_{max}$ *of on the* V_∞ *-sphere in the case* $\sin\psi = 0$

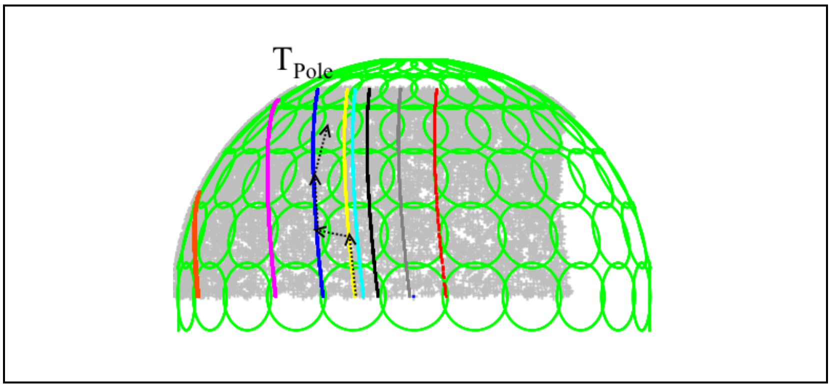

Fig. 3. The sequence of any solo GAMs aimed at increasing the spacecraft orbit inclination must approach the point T_{Pole} *. Resonant lines (lines of synchronisms between SC orbital period and GAM planetary orbital period) are {1:2, 3:4, 1:1, 5:4, 4:3, 3:2, 2:1, 3:1}*

The condition of the formation of a large inclination by jumping along mono-resonance (cranking gravity assists)

Let γ be the flight path angle of gravity-assist body orbit relative to central body at encounter. For the maximum of the spacecraft orbit inclination, the following conditions are necessary [1,3,7]:

$$\psi^* = \pi - \gamma,$$

$$\cos\rho = \cos\rho^* = \frac{V_\infty}{V_{pl}\cos\gamma}, \qquad (5)$$

$$\sin i_{\max} = \frac{V_\infty / V_{pl}}{\cos\gamma} = \frac{V_\infty}{\cos\gamma} = \cos\rho^*.$$

The $\rho - \psi$ -coordinates of the fixed point T_{res} corresponding to the extremum of the inclination at the fixed resonance line are presented for the specific main resonances in Table 1.

The rotation angles of $\mathbf{V}_\infty$ on one GAM near the terrestrial planets, corresponding to the required inclination of the orbit of the spacecraft are presented in Table 2.

Table 1: Coordinates ρ^*, ψ^* of the fixed point T_{res} corresponding to the extremum of the inclination for the specific main resonances

Resonance	ρ^*, grad	ψ^*, grad
1:1	75.5	180
3:4	62.7	180
4:3	85.8	180
5:4	83.6	180
3:2	89.3	180
1:2	34.0	180
2:1	83.0	0
3:1	75.5	0

Table 2: The rotation angles of $\mathbf{V}_\infty$ on one GAM near the terrestrial planets, corresponding to the required inclination of the orbit of the spacecraft

Planet	First cosmic speed V_{Fpl}, km/s	V_{pl}, km/s	φ_{max} for $i_{max} = 20°$, grad	φ_{max} for $i_{max} = 30°$, grad	φ_{max} for $i_{max} = 45°$, grad
Mercury	3.10	47.36	4.05	1.93	1.17
Venus	7.23	35.02	31.01	16.75	9.02
Earth	7.92	29.78	44.12	25.37	14.17
Mars	3.55	24.13	17.97	9.14	5.71

Analysis of the tables shows that jumps between the main resonances (pumping and cranking GAMs) when performing GAMs near Venus (which, according to Table 1, require a change of 9.8-13.3 degrees) are effective up to the values $i_{max} = 40°$. Otherwise, it is necessary to design a GAM mono-resonance sequence (only cranking GAMs). As Table 2 shows, any GAM near the planet Mercury is ineffective for performing hopping between resonances (the cells are marked dark). The same property is possessed by GAMs near Venus at $i_{max} > 40°$ and GAMs near the planet Mars at $i_{max} \geq 30°$ (only cranking GAMs are possible).

Using the presented tables, one can carry out a "calibration" of the V_∞ design value, "pushing" along its V_∞-sphere its pole of inclination T_{Pole} to the point T_{res} of maximum inclination on the line of the selected resonance.

Conclusion

The inclination pole coordinates on the invariant sphere of the asymptotic spacecraft velocity are derived. The procedure of a chain of GAMs leading to the inclination pole to achieve the geometrically feasible maximum of the inclination of the spacecraft orbit is analytically investigated. The characteristic "working" size of the spherical domain of an elementary GAM on the surface of the V_∞-sphere is determined. A finite covering of the connected track — a polygonal line from starting the GAM to the inclination pole — by spherical caps on the V_∞-sphere is constructed; this track can go along the resonance isolines and can jump between them. As a result, the formalized structure of the cranking GAMs makes it possible to automate the process of adaptive synthesis of bundles of the corresponding optimal trajectories.

As shown in the article the hopping between the main low-duration resonances when performing GAM near Venus (which require a change of 9.8-13.3 degrees) are effective up to the values $i_{max} = 40°$ (strong GAMs, pumping or cranking). Otherwise (also in most cases of "lite" Mars and every time – for Mercury) it is possible only design a GAMs according mono-resonance sequence (weak, i.e. cranking GAMs).

References

1. Grushevskii, A., Golubev, Yu.F., Koryanov, V., Tuchin, A. and Tuchin, D. "To the high inclined orbit formation with use of gravity assists", *Advances in the Astronautical Sciences*, Vol. 161, 2017, pp. 417-434.

2. Golubev, Yu.F., Grushevskii, A.V., Koryanov, V.V., Tuchin, A.G. and Tuchin, D.A. To the high inclined orbit formation with use of gravity assists maneuvers", *KIAM Preprint* No. 64, 2015 (Keldysh Inst. Appl. Math., Moscow, 2015).

3. Golubev, Yu.F., Grushevskii, A., Koryanov, V., Tuchin, A. and Tuchin, D. "The synthesis of gravity assist maneuvers sequence of the spacecraft to achieve high inclined orbits over ecliptic", *Journal of Computer and Systems Sciences International*, 2017, Vol. 56, No. 2, pp. 275–299. Pleiades Publishing, Ltd., 2017.

4. Labunsky, A.V., Papkov, O.V. and Sukhanov, K.G. *"Multiple Gravity Assist Interplanetary Trajectories"*, ESI Book Series (Gordon and Breach, London), 1998, pp. 9-266.

5. Pérez, J.M.S. "Trajectory Design of Solar Orbiter", *Proceedings of the 23rd ISSFD*, Pasadena, California, 2012. http://www.issfd.org/ISSFD_2012/ISSFD23_IMD1_3.pdf.

6. Buffington, B., Strange, N.J. and Campagnola, S. "Global moon coverage via hyperbolic flybys", *Proceedings of the 23rd ISSFD*, Pasadena, CA, 2012.

7. Strange, N.J., Russell, R. and Buffington, B. "Mapping the V-infinity globe", *Proceedings of the AIAA/AAS Space Flight Mechanics Meeting*, 2007. AAS Paper 07–277.

8. Kawakatsu, Y. "V∞ direction diagram and its application to swingby design", *Proceedings of the 21st International Symposium on Space Flight Dynamics*, Toulouse, France, 2009. http://issfd.org/ISSFD_2009/InterMissionDesignI/Kawakatsu.pdf.

9. Konstantinov, M. Petukhov, V. and Thein, M. "Optimization spacecraft insertion into the system of heliocentric orbits for Sun exploration", *Proceedings of the 65th International Astronautical Congress*, Toronto, Canada, 2014. IAC-14-C1.9.4.