Exploring the motion in libration point regions of perturbed three body problems Application to orbits in the Mars-Phobos system

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Abstract

Extensive literature has been published in the last decades concerning the motion in the so-called libration point regions, or the zones around the L1 and L2 equilibrium points, of the circular restricted three body problem ([1]). However, in some dynamical environments, the analysis of the motion in these regions cannot be performed only by means of the well-known methods used in the CR3BP (restricted 3-body problem). This is the case of planetary moons such as Phobos or Deimos, or of some binary asteroid systems, for instance. The objective of this paper is to present the numerical tools developed at CNES in order to explore the types of motions that can be derived from the natural dynamics in the vicinity of the libration points of strongly perturbed three body problems.

To begin with, existing methodologies for generating periodic libration point orbits (LPO) around L1 and L2 in the frame of the CR3BP will be shortly recalled. Then, the strategy implemented by the authors, aimed at introducing orbital and force model perturbations to the three body dynamics by means of numerical continuation techniques, will be presented. JAXA's MMX mission to the moons of Mars is regarded as a potential application case of these studies. The characteristics of the Mars-Phobos couple are quite unique in the Solar System. When designing trajectories in the vicinity of Phobos, the non-uniformity of the moon's shape and gravity field cannot be ignored. So, the effect of introducing the complex gravity of the moon, as well as its orbital eccentricity in the equations of motion, will be analyzed by starting at a "classical" LPO and applying the numerical tools described earlier in our paper. Moreover, the solutions obtained in this perturbed three body dynamics will be transformed into more realistic, full-ephemeris, dynamical models. Finally, preliminary station keeping strategies will be outlined, providing a first estimate of the operational cost and risk of using these trajectories as baseline locations for a future exploration mission.

Keywords: Lagrange, Halo, Periodic orbit, Mars, Phobos.

Introduction

The Circular restricted Three body problem (CR3BP) is a well known problem, where we study the motion of one massless object in the system of 2 bodies (called primaries) orbiting each-other, and describing circular orbits around their center of mass.
The equations that describe the motion of the massless object are the following:

$$\ddot{x} - 2\dot{y} = \frac{\partial \Omega}{\partial x}$$
$$\ddot{y} + 2\dot{x} = \frac{\partial \Omega}{\partial x}$$
$$\ddot{z} = \frac{\partial \Omega}{\partial z}$$

With:

$$\Omega = \frac{x^2 + y^2}{2} + \frac{1-\mu}{r_1} + \frac{\mu}{r_2}$$
$$\mathrm{r}_1^2 = (x-\mu)^2 + y^2 + z^2$$
$$\mathrm{r}_2^2 = (x-1+\mu)^2 + y^2 + z^2$$

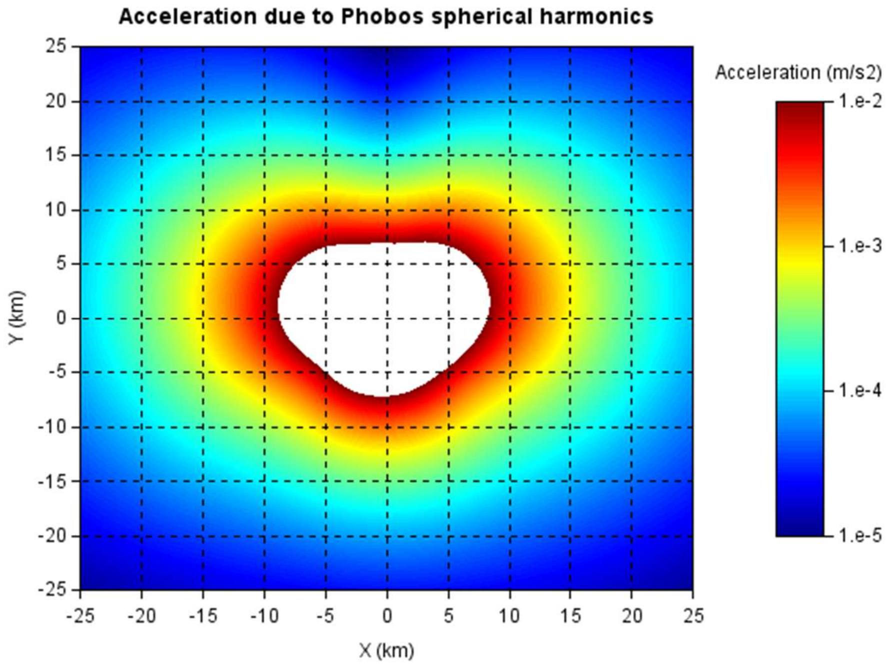
These equations are given in the barycencentric frame such that the x axis is aligned with the 2 primaries and the z axis is perpendicular to the orbit described by the 2 bodies. All quantities are adimensional (see Ref. [1] for instance for further details).

From these equations, five libration (that is equilibrium) points can be defined: L1... L5.

Our main goal is to define periodic (or nearly periodic) orbits around L1 or L2 in the Mars-Phobos system considering a realistic enough model.

The Mars-Phobos system differs significantly from other classical 2-body systems such as Earth-Moon or Sun-Earth/Moon, because the mass ratio for the Mars-Phobos system is very small (around 1.e-8) and the libration points are very close to Phobos (a few kilometers from the surface). And because the shape of Phobos is very irregular, the central force is not sufficient: spherical harmonics have to be taken into account.

This is illustrated by the following plot which shows the effects of spherical harmonics: norm of perturbing acceleration in the X-Y plane, where the X axis is along Mars-Phobos (Mars is towards the left) and Y axis is perpendicular and in Phobos' orbit plane. The potential is developed to degree and order 4. The date is arbitrary (1 June 2028). Of course, the value has no meaning inside Phobos body. At a distance of around 15 km from Phobos center, the amplitude of the perturbing acceleration can be up to 1×10^{-3} m/s^2. At the same distance, the central acceleration (due to Phobos) is around 3.e-3 m/s^2 (gravitational constant: 7.1×10^5 m^3/s^2, radius = 15×10^3 m). The relative effect is not negligible: the amplitude of the perturbation amounts to around 1/3 of the central attraction.



Another perturbing effect comes from the fact that the orbit of Phobos is not exactly circular. The eccentricity is not that high (around 1.5×10^{-2}), but induces a non-negligible effect. However, according to Ref [3] and [4], the effect is small compared to that due to spherical harmonics. Consequently, this effect will not be studied in this analysis.

Periodic orbits around L1/L2

We want to define orbits around the libration points L1 or L2 meeting the mission objectives and considering an accurate and realistic enough model.
By "accurate" we mean:

- Use of realistic ephemeris (typically from JPL)
- Complex gravitational fields (spherical harmonics to some degree / order)
- Additional representative forces taken into account if needed.

Defining orbits using such a model is the major objective of this paper.

Mars-Phobos system in more detail

Here are some characteristics of the Mars-Phobos system in comparison with Earth-Moon:

	Earth-Moon	**Mars-Phobos**
Distance (km)	384399	9400
Period (days)	27.3	0.3
Mass ratio (lightest / heaviest)	2.66×10^{-6}	1.65×10^{-8}
Distance L1 - surface (km) * Distance L2 - surface (km) *	57000 63000	3 3
Radius of smallest body (km)	1700	11

In comparison with the Earth-Moon system, some notable differences are:

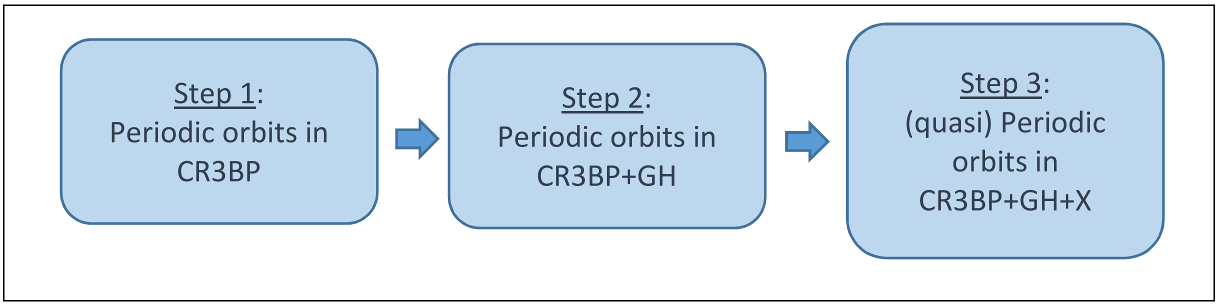
- The period: much smaller for Phobos (around 8 hours)
- The mass ratio: almost 150 times smaller for Phobos/Mars.

The libration points are equilibrium points of the dymamical system described by the equations shown above.
In particular L1 and L2 are the 2 libration points along line of the 2 primaries and at both sides of the smaller primary. The dynamics around these points in the CR3BP has been exhaustively studied in the past (see for instance Ref [1]).

For the Mars-Phobos system, the libration points L1 and L2 are found to be only a few km above the surface of Phobos (roughly 2 to 3 km). This means that the irregularities of the moon's gravity field will have a significant effect on the libration point orbits and their manifolds.

As already mentioned, the CR3BP model for orbits around Phobos is not well suited because of the irregularities of Phobos gravity field. The methodology we apply is the following (most simple physical model to the most complex one):

1. Computation of a LPO with the desired design parameters (size, period. . .), using the methods found in the literature (like the Lindstedt-Poincare procedures).
2. Computation of the dynamical substitute of the object computed in step 1) in a model taking into account the gravity field of Phobos, by means of a numerical continuation method. The continuation parameter, varying from 0 to 1, allows for a progressive inclusion of the acceleration caused by the gravity harmonics of Phobos.
3. Adjustment of orbits found at stage 2 in a full model, considering accurate ephemeris for Phobos in particular.

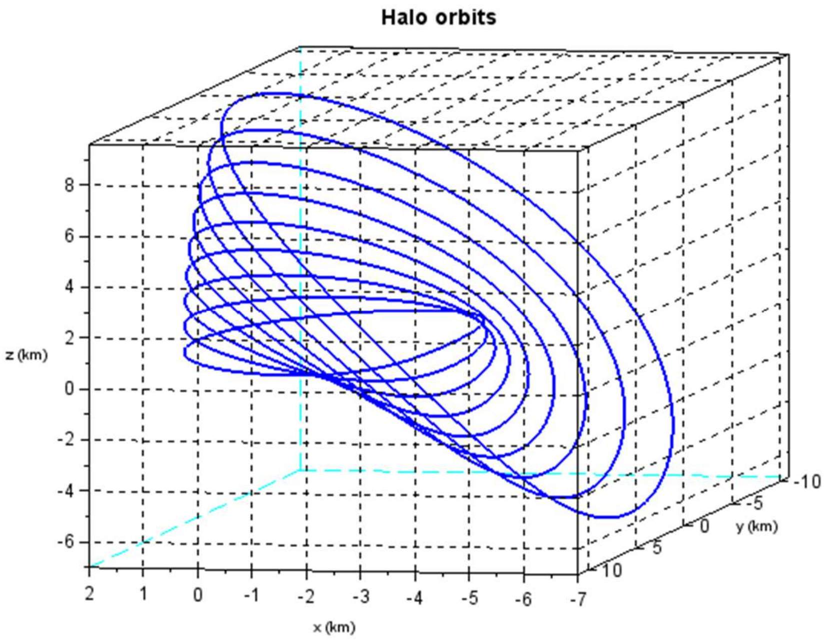


Periodic orbits in CR3BP

For the sake of simplicity and as a first application of the methodology, only periodic orbits around the libration points L1 and L2 have been considered, because the points are close to the surface.

More precisely, the periodic orbits that are studied are of 2 types: Halo (3D) and Lyapunov (2D, in the X-Y plane).

The following plot shows various Halo (and Lyapunov) orbits. The elongation around z (design parameter) is approximatively from 0 to $8x10^{-4}$. The origin of the frame is the Lagrange point (L2).



The dimensions (min/max value) along x, y, z are the following:

Approximate Z amplitude (adimensional)	X min/max (km)	Y max/max (km)	Z min/max (km)
~0	-2.7 1.9	-7.4 7.4	-0.0 0.0
$2x10^{-4}$	-2.9 1.9	-7.6 7.6	-1.7 2.1
$4x10^{-4}$	-3.5 1.8	-8.3 8.3	-3.4 4.4
$6x10^{-4}$	-4.5 1.6	-9.4 9.4	-5.0 6.7
$8x10^{-4}$	-6.0 1.1	-10.8 10.8	-6.5 9.5

The “Approximate Z amplitude” is the design parameter used to compute the solutions.

CR3BP + gravity harmonics (CR3BP+GH)

It happens that the Phobos performs one revolution around Mars in the time that it performs one revolution around its rotation axis. This simplifies the problem: the potential of the perturbing force can be considered constant in the rotating frame.

The new equations considering the effect of the additional force are then derived simply from the equations in the CR3BP system:

$$\ddot{x} - 2\dot{y} = \frac{\partial \Omega_{tot}}{\partial x} = \frac{\partial \Omega}{\partial x} + \gamma_x$$

$$\ddot{y} + 2\dot{x} = \frac{\partial \Omega_{tot}}{\partial y} = \frac{\partial \Omega}{\partial x} + \gamma_y$$

$$\ddot{z} = \frac{\partial \Omega_{tot}}{\partial z} = \frac{\partial \Omega}{\partial z} + \gamma_z$$

where $\gamma_x, \gamma_y, \ \gamma_z$ are the (adimensional) acceleration due to the perturbation.

An important remarks is that the quantity $\frac{V^2}{2} - \Omega_{tot}$ is constant (and is equal to the Jacobi constant divided by -2).

Numerical continuation

This method is used to compute a trajectory for a complex model, knowing the trajectory in a simpler one. In our application, the trajectory in known in CR3BP model, and we would like to derive a trajectory in CR3BP+GH.

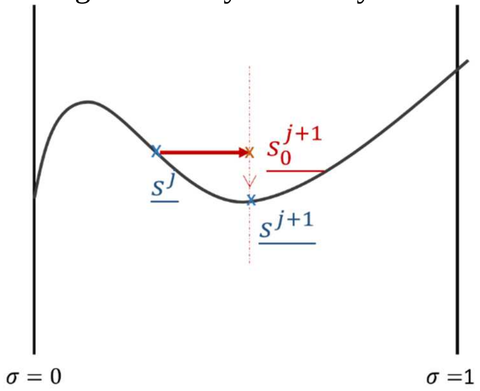
Let's assume that the complex model is obtained from the simpler one by adding one perturbation, weighted with the quantity σ.

When σ is 0 => the model is the simple model (for which a solution is known).

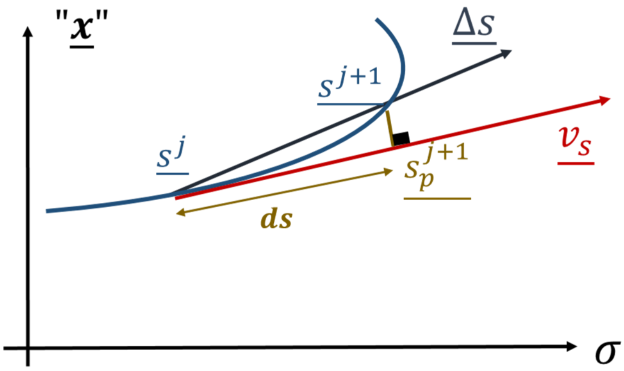
When σ is 1 => the model is the complex model (for which the solution is unknown).

The idea is to look for a solution iteratively, starting from σ = 0 and increasing the value progressively up to σ = 1.

The fist idea called "natural scheme" consists in incrementing σ by a constant value at each step, and the result of each step is used as the initial guess for the next one, unless the algorithms does not converge in which case the value of σ is reduced. The algorithm stops when σ >= 1. The algorithm may not always converge particularly in case of bifurcations.



The second method used is called "quasi arc length". The initial guess is looked for in the direction of the tangent of the curve (x, σ) where x is the solution to the problem. With this method σ is not constant.



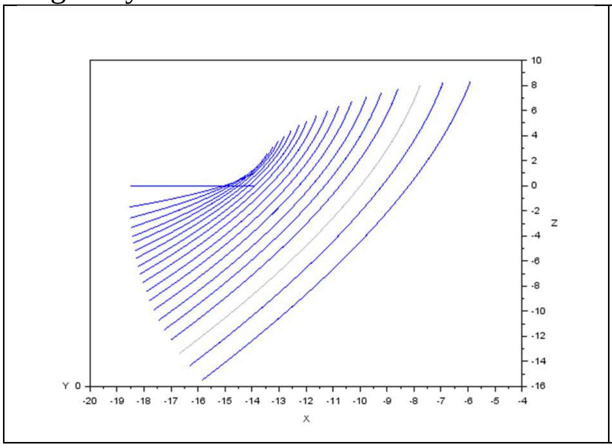
Periodic orbits in CR3BP+GH

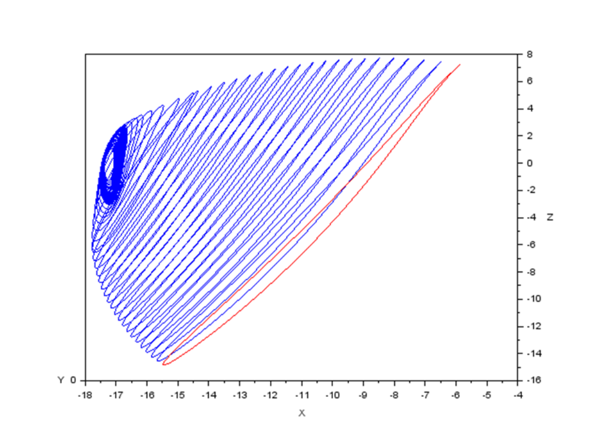
Before generating periodic in the CR3BP+GH system, let's take a look at some differences with CR3BP regarding the positions of the equilibrium points.

	L1 Coordinates in km	L2 Coordinates in km
CR3BP	16.561, 0, 0 (norm = 16.561)	-16.582, 0, 0 (norm = 16.580)
CR3BP+GH	17.326, -0.358, -0.202 (norm = 17.331)	-17.241, -0.358, -0.103 (norm = 17.245)

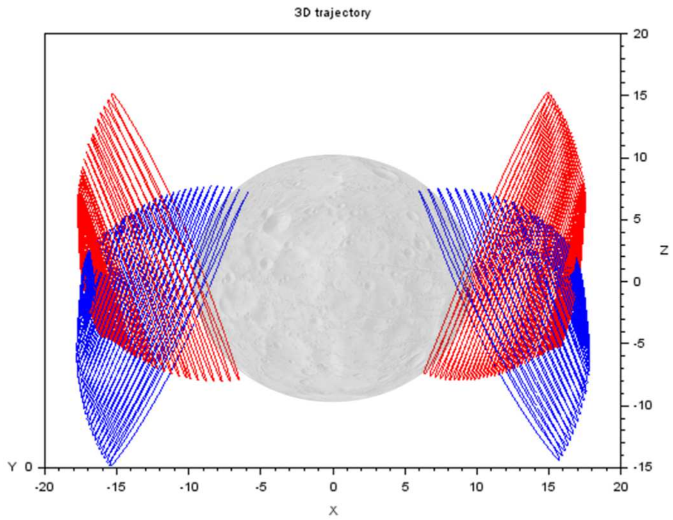
The difference between CR3BP and CR3BP+GH are not that big: max 5% on the components of the vector "Phobos" -> Equilibrium point.

Applying the quasi arc length continuation method, periodic orbits can be computed considering the gravity harmonics:





The solutions in CR3BP are shown in the figure below where Phobos is also represented which gives an idea of their size.



As an example for L2, the following table compares the periodic orbits for the 2 models:

CR3BP				CR3BP+GH			
Ax	Ay	Az	T	Ax	Ay	Az	T
9.917	26.96	23.772	3.57	9.884	24.523	22.057	3.27
6.08	18.886	11.802	3.74	4.617	14.454	14.647	3.58
3.189	10.192	0.0	3.73	1.568	4.963	7.638	3.61
2.249	7.196	0.0	3.72	0.878	2.778	5.715	3.61
1.008	3.229	0.0	3.70	0.347	1.144	2.608	3.60

Ax, Ay, Ay are the amplitudes in km. T in the period in hours.

Some results have in fact been obtained using another method: continuation on energy, so that a larger set of orbits can be obtained.
The conclusion is that the shape has significantly changed, in particular for the smaller orbits. We notice that the amplitude along z which is 0 in the CR3BP case, is not 0 in the CR3BP+GH one.

Fitting a LPO CR3BP+GH to a more realistic dynamical model

Objectives, principles

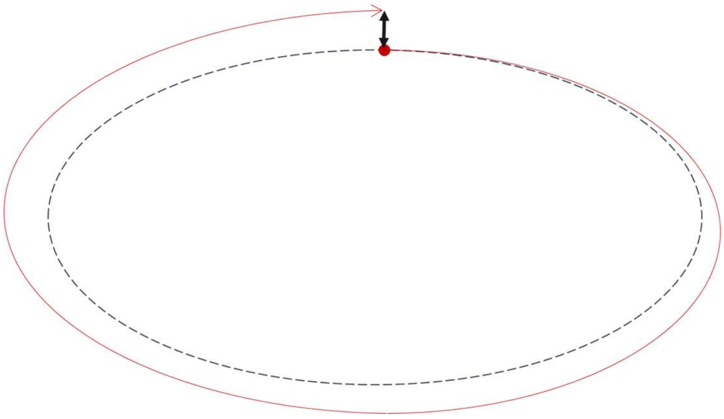
Our next and final goal is to adapt the CR3BP+GH orbits to a realistic model, that takes the effect of additional forces (possibly) and realistic body ephemerides into account.

There are several aspects that have to be handled.

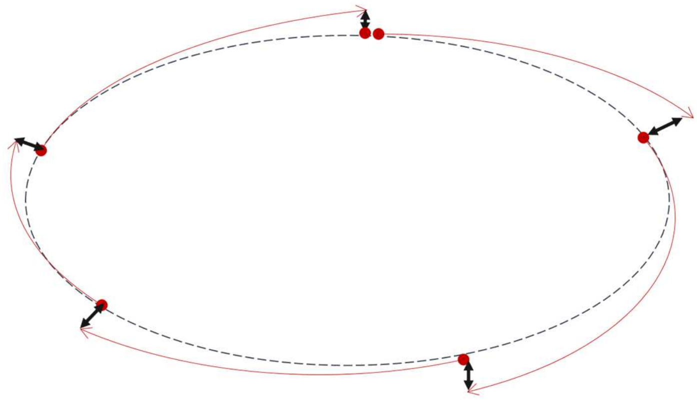
First, the reference trajectory (CR3BP or CR3BP+GH model) has to be adapted to the “real world”, that is un-normalized. The best way to “un-normalize” the trajectory is to use a mean value for the angular velocity vector and an instantaneous value (that is function of time) for the distance between the 2 bodies. The resulting trajectory cannot be obtained by propagation from the initial state, but is supposed to be close enough to a “real” trajectory.

The second step consists in adjusting the trajectory obtained after step 1. To do this, 2 methods have been envisaged and used: single shooting or multiple shooting.

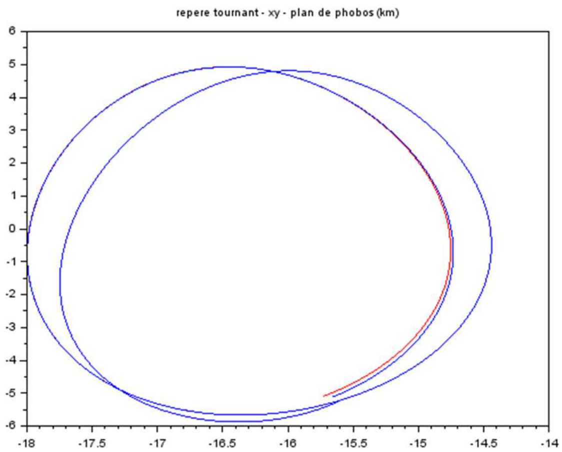
With the single shooting method, only the initial state vector is adjusted. The fitting consists in minimizing the “measurements” (position and/or velocity vector) residuals by a least squares method. But if the initial state vector is too far from the solution, or if the arc is too long, the algorithm may not converge.



The multiple shooting method enable to fit a trajectory simultaneously over several arcs. We start from the reference trajectory and adjust each intermediate position and velocity vectors by minimizing gaps. For more information about this method, see Ref [2].



After adjustment, the orbit can be propagated. But the difference with the adjusted orbit remains small for about 2 orbit periods.



Application to Phobos exploration

This part aims at evaluating how the orbits that have been found can be used for a “real” mission. Two aspects are considered: stability of the orbit and station keeping (computation of orbit corrections).

Station keeping

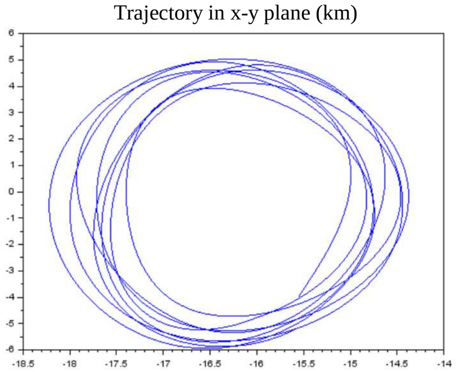
We would like to design an orbit for an arbitrarily long period that could be used by some mission. This trajectory is supposed to contain maneuvers (velocity increments).

As we have seen that propagating for 2 orbit periods works well (after adjustment), we consider applying maneuver increments every 2 orbit periods. What has been implemented is based on what has already been shown:

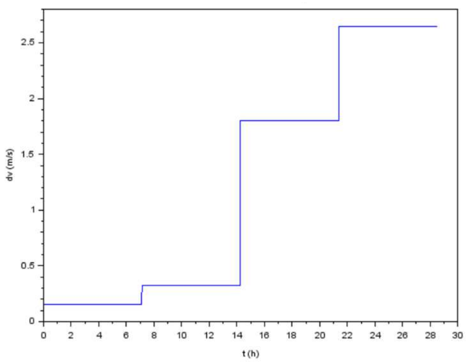
- Perform multiple shooting over 2 orbit periods (this changes the initial state) with a reference orbit consistent with CR3BP+GH (see previous paragraph).
- Propagate the orbit from the initial state for 2 orbit periods (the final initial state is x1 at final instant t1)
- Retart the same process on the next arc (starting at t1), the multiple shooting process only changing the initial position (that is position at t1) and not the velocity. The velocity increment at instant t1 can then be computed.
- And so on…

The result is rather satisfying. The following illustration shows the result over 30 hours, performing maneuvers every 8 hours. The total ΔV is only 2.6 m/s. If this phase was to be extended over one year, the cost would be about 760 m/s.

Trajectory in x-y plane (km)



Velocity increments (m/s)



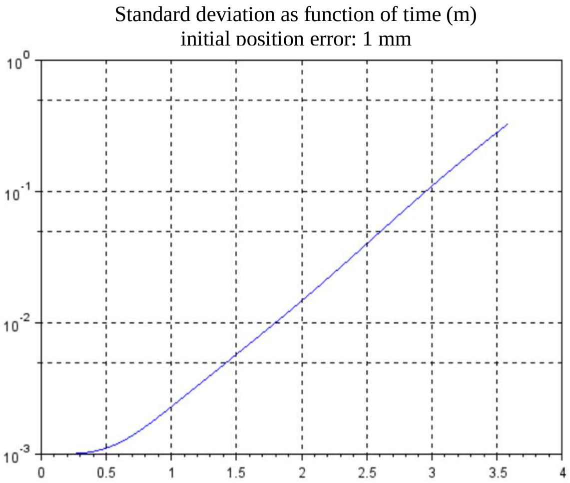
Of course this is not really "station keeping" in the sense that all perturbing effect (forces such as SRP, maneuver execution errors) are not taken into account, but at least a reference trajectory that includes maneuvers can be built.

Propagation of covariance

How the orbit is stable can be measured by a covariance propagation.

We assume some error (1σ) on the initial position (same error on each component) and no error on the velocity at the same time.

When propagating we observe that the error is multiplied by about 1000 in 4 hours (which corresponds to about 1 orbit period). It means that if we start with an error of 1 meter (which is very small), we end up with an error of 1 km after 4 hours, that is far from the reference trajectory.



It also means that the orbit would have to be corrected more often than once every 4 hours. This is not very good news. If no autonomous measurements are available, this does not seem feasible.

Summary and conclusion

The paper has presented the main results of the work performed on LPOs in the Mars-Phobos system. Starting from periodic (Halo) orbits in the classical CR3BP problem, periodic orbits have first been computed taking into account the complex gravity field of Phobos. These periodic orbits have then adjusted using a more complex model (realistic ephemeris in particular).

Adjustment over successive arcs showed that it is possible the build a reference trajectory that includes maneuvers over a long enough period of time. The bad news is that the trajectory is very sensitive to initial conditions (as could be expected). Four hours seems to be the maximum time without maneuvers (assuming a good initial position accuracy). Hence it does not seem possible to design a mission in the vicinity of the Lagrange points that would last longer than a few hours, unless in-situ autonomous measurements are available.

References

1. G. Gomez, A. Jorba, J. Masdemont, and C. Simo. “Dynamics and Mission Design near Libration Points”, volumes 1 to 4. World Scientific, Singapore, 2001.

2. J. Stoer, R. Bulirsch, “Introduction to Numerical Analysis”, second edition, Springer-Verlag.

3. M. Zamaro, PhD. Thesis “Natural and artificial orbits around the Martian moon Phobos.” Department of Mechanical and Aerospace Engineering, University of Strathclyde, 2014-2015.

4. E. Canalias, L. Lorda, T. Martin, J. Laurent-Varin, J.C. Marty, Y. Mimasu “Trajectory analysis for the Phobos proximity phase of the MMX mission”, conference paper ISSFD 2017, Matsuyama (Japan).