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Optimization of Multiple-Impulse Perturbed Cooperative Rendezvous for Spacecraft

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Abstract

This paper develops a hybrid trajectory optimization method for cooperative space rendezvous using impulsive thrust. With propellant constraints considered, a two-step multiple-impulse optimization strategy including two-sided feasible and infeasible iteration is employed. Numerical results show that the total propellant consumed in the cooperative manner is always but not necessarily less than that in the uncooperative manner. But for the chasing spacecraft, the cooperative manner can save energy to finish a rendezvous mission.

Keywords: trajectory optimization, cooperative rendezvous, propellant constraints.

Introduction

Rendezvous and docking, as a significant space mission, has been researched in many ways, e.g. the automated rendezvous [1], the autonomous rendezvous [2], multiple-impulse multiple-resolution rendezvous [3], and robust rendezvous [4]. Most previous researches are based on the uncooperative scenarios where an active spacecraft approaches a passive target. However, this uncooperative manner may be inferior to the counterpart, the cooperative manner, in terms of the consumption of propellant or time. Some efforts have been paid on the cooperative rendezvous with continuous thrust. Coverstone-Carroll and Prussing [5-6] presented analytical solutions for cooperative power-limited rendezvous in the linearized gravity field and further extended the theory to inverse-square gravity field. Feng, et al. [7] researched the far-distance rapid cooperative rendezvous. Comparatively, researches on impulsive cooperative rendezvous are inadequate. Prussing and Conway [8] studied the optimal terminal maneuver for a cooperative impulsive rendezvous. Mirfakhraie and Conway [9] provided a method of determining fuel-optimal trajectories for impulsive cooperative rendezvous within fixed-time. Dutla and Tsiotras [10] provided the solution of cooperative rendezvous analytically via Hohmann-Hohmann and Hohmann-Phasing. In this paper, the hybrid optimization approach for cooperative multiple-impulsive rendezvous with propellant constraints are studied, with the effects of non-spherical gravity and the atmosphere drag considered. Specifically, a two-sided feasible iteration optimization model [11] is first formulated to locate the unperturbed solution which is solved by differential evolution (DE) algorithm. Then, the infeasible model is employed to obtain the perturbed solution via sequential quadratic programming (SQP). The rest of the paper is organized as follows. Section II describes the multiple-impulse cooperative rendezvous problem. Section III presents the hybrid optimization method to solve the problem, followed by Section IV where detailed simulation results are shown. Finally, the conclusion is drawn in Section V.

Multiple-Impulse Cooperative Rendezvous Problem

In the cooperative rendezvous, both the target spacecraft and the chaser spacecraft phase actively via multiple impulsive maneuvers. The motion of the two spacecraft is govern by following dynamic equations:

$$\ddot{\boldsymbol{r}} = -\frac{\mu}{r^3}\boldsymbol{r} + \boldsymbol{a}_{\text{nonspherical}} + \boldsymbol{a}_{\text{drag}} + \boldsymbol{a}_{\text{other}} \tag{1}$$

where $\boldsymbol{r}$ is the position vector; μ is the geocentric gravitational constant; $\boldsymbol{a}_{\text{nonspherical}}$, $\boldsymbol{a}_{\text{drag}}$, $\boldsymbol{a}_{\text{thrust}}$, and $\boldsymbol{a}_{\text{other}}$ are the accelerations caused by the non-spherical gravity, the atmosphere drag, the thrust, and other factors, respectively. After ith (jth) intermediate impulse for chaser (target), the state vectors vary accordingly as

$$\begin{cases} \boldsymbol{r}_i^{\text{cha}+} = \boldsymbol{r}_i^{\text{cha}-} = \boldsymbol{r}_i^{\text{cha}}\ ,\ \boldsymbol{r}_j^{\text{tar}+} = \boldsymbol{r}_j^{\text{tar}-} = \boldsymbol{r}_j^{\text{tar}}, i = 1,\ 2,\ ..., n \\ \boldsymbol{v}_i^{\text{cha}+} = \boldsymbol{v}_i^{\text{cha}-} + \Delta\boldsymbol{v}_i^{\text{cha}}, \boldsymbol{v}_j^{\text{tar}+} = \boldsymbol{v}_j^{\text{tar}-} + \Delta\boldsymbol{v}_j^{\text{tar}}, j = 1,\ 2,\ \ldots, m \end{cases} \tag{2}$$

where the superscript ‘cha’ and ‘tar’ denote the chaser and the target, respectively; n and m are their total impulse number; $\Delta\boldsymbol{v}_i^{\text{cha}}$ and $\Delta\boldsymbol{v}_j^{\text{tar}}$ represent their impulse vectors; the superscript ‘-’ and ‘+’ denote the states before and after one impulse is applied, respectively. Provided that $\boldsymbol{r}(t+\Delta t) = \boldsymbol{f}[\boldsymbol{r}(t), \boldsymbol{v}(t), t, t+\Delta t]$ and $\boldsymbol{v}(t+\Delta t) = \boldsymbol{g}[\boldsymbol{r}(t), \boldsymbol{v}(t), t, t+\Delta t]$ are the solution of Eqn 1, the state vectors of two adjacent impulses will have relationship as

$$\begin{cases} \boldsymbol{r}_i^{\text{cha}} = \boldsymbol{f}(\boldsymbol{r}_{i-1}^{\text{cha}}, \boldsymbol{v}_{i-1}^{\text{cha}+}, t_i^{\text{cha}} - t_{i-1}^{\text{cha}}), \boldsymbol{r}_j^{\text{tar}} = \boldsymbol{f}(\boldsymbol{r}_{j-1}^{\text{tar}}, \boldsymbol{v}_{j-1}^{\text{tar}+}, t_j^{\text{tar}} - t_{j-1}^{\text{tar}}) \\ \boldsymbol{v}_i^{\text{cha}-} = \boldsymbol{g}(\boldsymbol{r}_{i-1}^{\text{cha}}, \boldsymbol{v}_{i-1}^{\text{cha}+}, t_i^{\text{cha}} - t_{i-1}^{\text{cha}}), \boldsymbol{v}_j^{\text{tar}-} = \boldsymbol{g}(\boldsymbol{r}_{j-1}^{\text{tar}}, \boldsymbol{v}_{j-1}^{\text{tar}+}, t_j^{\text{tar}} - t_{j-1}^{\text{tar}}) \end{cases} \tag{3}$$

where t_i and t_j are the time sequence of impulses executed by the chaser and the target. The goal of their phasing is to attain the same position and velocity, i.e., $\boldsymbol{r}_f^{\text{cha}} = \boldsymbol{r}_f^{tar}, \boldsymbol{v}_f^{\text{cha}} = \boldsymbol{v}_f^{tar}$ where the subscript ‘f’ denotes the terminal state of the variable at $t = t_f$. And the phasing optimization strategy to determine the optimal time and optimal position of each impulse for a certain objective J, e.g. the least total velocity increment:

$$\min J = \Delta v^{\text{cha}} + \Delta v^{\text{tar}} = \sum_{i=1}^{n} \| \Delta v_i^{\text{cha}} \| + \sum_{j=1}^{m} \| \Delta v_j^{\text{tar}} \| \tag{4}$$

In more realistic situation, the impulse time is constrained by

$$\begin{cases} t_i^{\text{cha}} - t_{i-1}^{\text{cha}} \geq \Delta T^{\text{cha}},\ t_j^{\text{tar}} - t_{j-1}^{\text{tar}} \geq \Delta T^{\text{tar}},\ i = 1, 2, \ldots, n; \\ t_i^{\text{cha}} \in [t_0, t_f],\ t_j^{\text{tar}} \in [t_0, t_f], \quad j = 1, 2, \ldots, m \end{cases} \tag{5}$$

where t_0 and t_f are the initial time and terminal time of the rendezvous mission; ΔT is the minimum time interval between two adjacent impulses.

Hybrid Optimization Method

The parameter optimization methods to the above multiple-impulse rendezvous problem can be divided into two types, feasible iteration approach and infeasible iteration approach. The detailed introduction of the two methods can refer to Ref. 11. In this paper, the previous two-step optimization method [3] for uncooperative rendezvous is improved for coping with the cooperative rendezvous problem. First, a feasible iteration model including target spacecraft's manoeuver is formulated. The differential evolution (DE) algorithm are used to solve unperturbed solution. Then the infeasible iteration model is employed to obtain the perturbed solution via the sequential quadratic programming (SQP).

Feasible Iteration for Unperturbed Solution

At this stage, the dynamics is simplified as $\ddot{\boldsymbol{r}} = -\boldsymbol{r} \cdot \mu / r^3$ for efficient solution. The designed variables are $\boldsymbol{X} = [t_i^{\text{cha}}, \Delta v_{ix}^{\text{cha}}, \Delta v_{iy}^{\text{cha}}, \Delta v_{iz}^{\text{cha}}, t_j^{\text{tar}}, \Delta v_{jx}^{\text{tar}}, \Delta v_{jy}^{\text{tar}}, \Delta v_{jz}^{\text{tar}}]$ i = 1, …, n-2, j = 1,…, m which includes the impulse time t_i^{cha} and t_j^{tar}, the first n-2 impulse vectors for the chaser and the m impulse vectors for the target. Then the total numbers of optimization variables will be $4(n+m)-6$. The state vectors of the chaser (target) evolve from a certain impulse time t_{i-1} (t_{j-1}) to next impulse time t_{i-1} (t_{j-1}) following the relationship in Eqn 3. The last two impulses of the chaser are computed by a Lambert algorithm [12] to automatically satisfy the terminal rendezvous constraint.

$$\begin{cases} (\boldsymbol{v}_{n-1}^{\text{cha}+}, \boldsymbol{v}_n^{\text{cha}-}) = Lambert(\boldsymbol{r}_{n-1}^{\text{cha}}, \boldsymbol{r}_n^{\text{cha}}, t_n^{\text{cha}} - t_{n-1}^{\text{cha}}) \\ \Delta \boldsymbol{v}_n^{\text{cha}} = \boldsymbol{v}_n^{\text{cha}+} - \boldsymbol{v}_n^{\text{cha}-}, \Delta \boldsymbol{v}_{n-1}^{\text{cha}} = \boldsymbol{v}_{n-1}^{\text{cha}+} - \boldsymbol{v}_{n-1}^{\text{cha}-} \end{cases} \tag{6}$$

where *Lambert* denotes the Lambert function; $\Delta \boldsymbol{v}_n^{\text{cha}}$ and $\Delta \boldsymbol{v}_{n-1}^{\text{cha}}$ denote the last two impulses. Then the Differential Evolution (DE) algorithm is employed to solve this optimization problem for unperturbed solution. The details for the DE algorithm can refer to Ref. 13.

Infeasible iteration for Perturbed Solution

For the perturbed solution, the propagation function requires to be rectified for fulfilling the high-fidelity numerical trajectory propagation. Compared with the feasible iteration, the Lambert algorithm is unemployed for rendezvous conditions, which are further satisfied upon the convergence of numerical optimization algorithm, i.e., SQP. The SQP is known as an effective algorithm handle nonlinear constraints.

Numerical Results

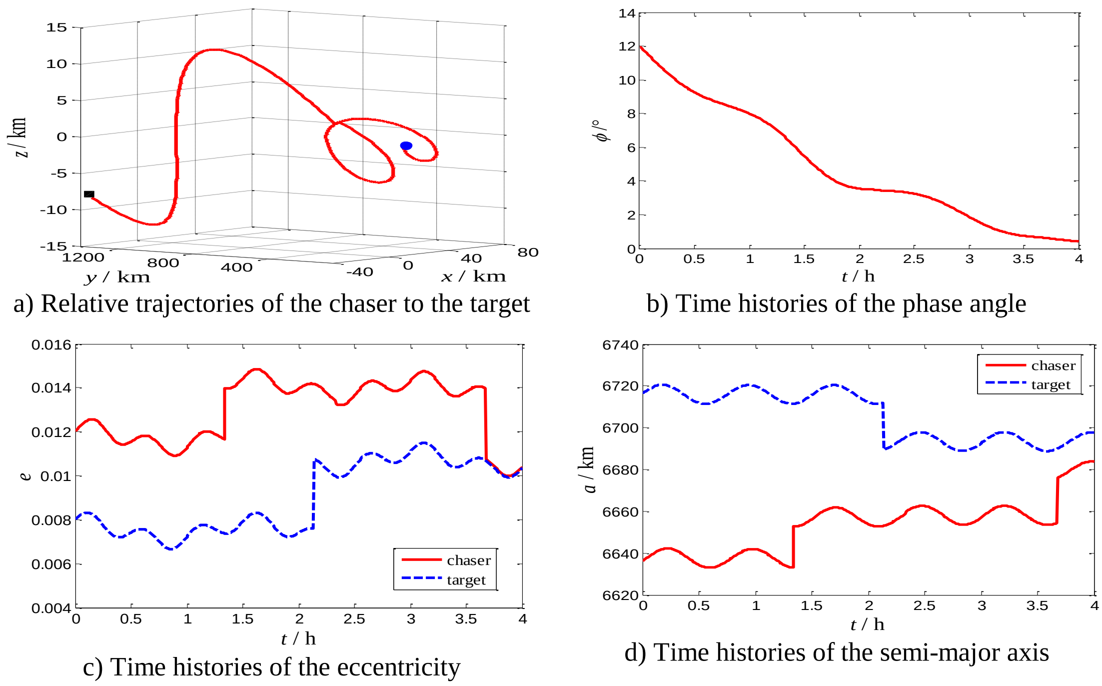
In this section, we further test the hybrid optimization approach by a practical four hour cooperative rendezvous mission. The initial Gregorian universal coordinated time of the mission is Aug. 17 2016 10:51:22. The initial states of two spacecraft are $\boldsymbol{E}^{\textbf{tar}}$ = (6716.3 km, 0.008, 42.86°, 55.75°, 127.49°, 10°) and $\boldsymbol{E}^{\textbf{cha}}$ = (6636.1 km, 0.012, 42.84°, 55.92°, 125.48°, 0°), which are expressed by the classical osculating orbital elements in this order: semi-major axis, eccentricity, inclination, right ascension of ascending node, argument of perigee, true anomaly. The aimed terminal relative position and velocity of the chaser are $\boldsymbol{\rho}_{aim}$ = [-13.5, -50.0, 0]^T (km) and $\dot{\boldsymbol{\rho}}_{aim}$ = [0, 23.23, 0]^T (m/s), which are described in the target's local vertical-local horizontal (LVLH) frame [5]. The maximum terminal orbit eccentricity of the target and the chaser is 0.012. The maximum tolerance of the terminal relative distance and velocity is $\|\boldsymbol{\delta}_r\| = 50\text{m}$ and $\|\boldsymbol{\delta}_v\| = 0.05\text{m/s}$.

Optimization Results

According to the input provided above, the cooperative rendezvous problem is successfully solved with different numbers of impulses of the chaser and target using the proposed approach. Ten independent runs are carried for each case, and the statistical results are provided in Table 1. To illustrate more minutely, the detailed precise maneuver plan for a "three-to-one" case is presented in Table 2, in which the numbers of impulses of the chaser and target are three and one, respectively.

Table 1: Optimization results with different numbers of impulses

Number of impulses		Total velocity increment (m/s)				Success
Chaser	Target	Best	Worst	Average	SD	rate (%)
2	1	41.595	42.263	42.095	0.2511	100%
2	2	41.885	41.939	41.896	0.022	100%
3	1	40.130	42.236	41.605	0.749	100%
3	2	40.695	42.235	41.694	0.534	100%
3	3	41.625	41.699	41.653	0.027	100%
4	1	41.850	42.235	41.963	0.139	100%
4	2	41.637	42.235	41.991	0.239	100%



a) Relative trajectories of the chaser to the target

b) Time histories of the phase angle

c) Time histories of the eccentricity

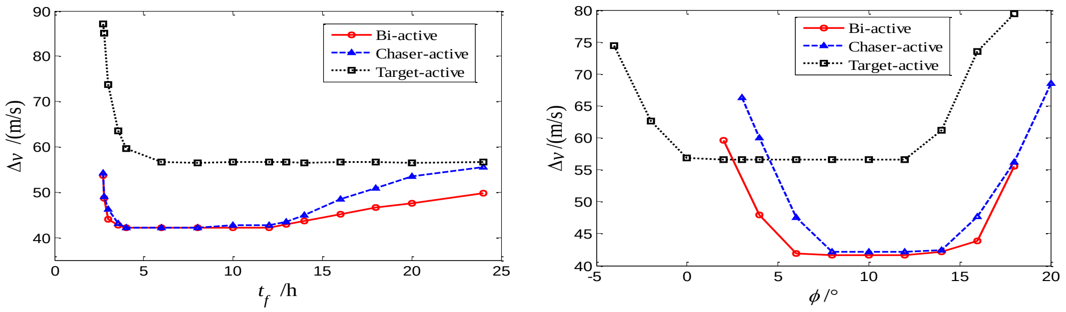
d) Time histories of the semi-major axis

Fig. 1: The results of the best solution in "three-to-one" case

According to Table 1, the success rate for each case is 100% and the corresponding standard deviation is quite small, which indicates that the proposed optimization approach is effective and robust to obtain a near-optimal solution of the multiple-impulse cooperative rendezvous problem. The minimum velocity increment obtained is 40.13 m/s, which is executed by both the chaser and the target. In the best solution's maneuver plan, the impulse sequence of the chaser is $[t, \Delta \boldsymbol{v}_1]$ = [4806.05, -10.52, -6.60, -3.67], $\Delta \boldsymbol{v}_2$ = [8382.04, -0.16, 0.49, 0.24], $\Delta \boldsymbol{v}_3$ = [13238.14, 5.49, 10.31, 5.50] and that of target is $\Delta \boldsymbol{v}_1$ = [7698.36, -7.21, -10.28, -5.48]. The total velocity increment required by the chaser and the target is 26.43 m/s and 13.7 m/s, respectively. In order to validate the convergence of this best solution, the results are shown in Fig. 1. Specially, the three dimensional relative trajectories of the chaser to the target are shown in Fig. 1(a), and the time histories of the phase angle, eccentricity as well as semi-major axis of the two spacecraft are shown in Fig. 1(b), Fig. 1(c) and Fig. 1(d). In Fig. 1(a), the terminal relative position and velocity of the chaser to the target is $\boldsymbol{\rho}_{\text{aim}}$ = [-13499.98, -50000.36, -0.05]^T (m) and $\dot{\boldsymbol{\rho}}_{\text{aim}}$ = [-0.0001, 23.2299, 0.0002]^T (m/s) respectively, which primly coincide the predetermined aimed relative state conditions. It can also be found in Fig. 1(c) that the terminal eccentricity of the two spacecraft are approximately 0.01, which can ensure that their terminal orbits are near circular.

Comparisons with uncooperative rendezvous

In order to compare the cooperative rendezvous with uncooperative rendezvous, total velocity increment of the target-active and bi-active manners in different situations are also recorded and shown in Fig. 2.



a) Velocity increment vs. rendezvous duration. b) Velocity increment vs. the initial phase angle.

Fig. 2: The results of the best solution in "three-to-one" case

Fig. 2 (a) shows that the total velocity increment of the chaser-active manner and the bi-active manner is nearly the same within a short mission duration. But as the duration increases, the total velocity increment of the bi-active manner is evidently shorter than chaser-active manner, which demonstrates the advantages of the cooperative rendezvous in propellant consumption. The pure target-active manner costs most propellants in three manners. From Fig. 2 (b), the total velocity increment of the bi-active manner and the chaser-active manner are almost the same when the initial phase angles are within a range of 8° to 14°. When the angles are out of the range, the bi-active manner requires less velocity increment. In terms of the pure target-active rendezvous, the velocity increment required is most in three manners. Thus, in terms of the total velocity increment, the cooperative rendezvous may be usually but not necessarily superior to the uncooperative rendezvous strategy.

Conclusions

In this paper, a two-step trajectory optimization method for cooperative rendezvous using impulsive thrust is developed. It is demonstrated that the method is effective and robust, with a successful solving rate of 100%. Finally, the velocity increment required in the rendezvous mission between the cooperative manner and the non-cooperative manner is compared. According to the results, we understand that the total velocity increment of the cooperative rendezvous is always but not necessarily less than that of the non-cooperative rendezvous for different rendezvous scenarios, and it mainly depends on the initial phase angle and the duration of the rendezvous mission. Nevertheless, we found that the chasing spacecraft in the cooperative rendezvous always costs less propellant than that in the active-passive rendezvous. Thus, the cooperative rendezvous should be of significance to a mission when the chasing spacecraft is lack of propellant.

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