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Review of the Draper Semi-analytical Satellite Theory (DSST)

P. Cefola [1], J. F. San-Juan [2], S. Setty [3] and R.Proulx [4]

[1] *Adjunct Professor, Department of Mechanical and Aerospace Engineering, State University of New York at Buffalo, Amherst, NY, USA; also Consultant in Aerospace Systems, Spaceflight Mechanics, and Astrodynamics, Vineyard Haven, MA, USA*
[2] *Scientific Computing Group (GRUCACI), Universidad de La Rioja, ES-26006 Logroño, Spain*
[3] GMV@ESA, *Darmstadt, Germany*
[4] *Consultant in Aerospace Systems, Spaceflight Mechanics, and Astrodynamics, Newton, MA, USA*

Abstract

The DSST includes comprehensive models for both the mean-element and the short-periodic motions of an artificial Earth satellite orbit. These models are developed in the equinoctial orbital elements via Perturbation Theory, in particular the classical Method of Averaging. The development assumes a near identity transform to the osculating elements from the mean elements and an expansion for the mean element equations of motion. The constants in the DSST are the mean equinoctial elements at an arbitrary epoch. The DSST includes the partial derivatives of the osculating elements at an arbitrary output time with respect to the epoch values of the mean equinoctial elements and the dynamical parameters. The partial derivative calculation includes variational equations constructed from the mean element equations of motion.

Keywords: semi-analytical theory, mean equinoctial elements, short-periodic motion, partial derivatives, orbit estimation, long-term orbit.

Introduction

DSST is a semi-analytical orbit propagator, whose original Fortran 77 version exists in two forms, one as an option within the Massachusetts Institute of Technology (MIT) version of the Goddard Trajectory Determination System (GTDS) computer program [1] and the other as a standalone orbit propagator package [2]. DSST has also recently been implemented in the open source Orekit Flight Dynamics library (Java) [3], and in the C/C++ version of the DSST Standalone (currently in testing) [4].

The evolution of DSST Standalone is described in Ref. 2. The development of the DSST Standalone started in 1983. Later, between 1990 and 1994, it was modified by the Draper Laboratory technical staff to be used as the orbit propagator in the Mission Support Program for both the Landsat 6 and Radarsat-1 missions [2, 5]. The design of this version is described in detail in Ref. 6. The main contribution of the early 1990s focused on the top-level interface, which was reimplemented to be file driven.

During the late 1990s, additional improvements were made to DSST Standalone, including:

- Updates to improve the maintainability of the source code, including the replacement of common blocks with structured records and modules. This improvement implies unique variable names, which helps avoid confusion within the subroutines.
- Solar and Lunar solid-Earth tide model in the mean element equations of motion.
- Integration using the FK5 (J2000.0) coordinate frame.

- Geopotential size expanded to a degree and order of 50.
- Short-periodic tesseral linear combination terms.
- Short-periodic lunar-solar terms.

Semi-analytical propagators combine the characteristic speed of analytical propagation and the accuracy of numerical propagation. They are based on the concept of expressing the orbital state as the addition of two components: one with slow dynamics and other that collects the fast variations. The first one includes the secular and long-period terms, thereby it represents a centered or mean model. As its rate of change is slow when compared to the orbital period, a numerical integration can be performed with very long steps, which implies fast computation. The other component, the fast one, collects the short-period terms, hence it represents the difference between the osculating and the mean parameters. This component is computed only at the final epoch, in order to recover the osculating ephemeris from the mean values.

The necessary equations to deal with both components can be derived from asymptotic expansions, both for the mean-element equations of motion and for the short periodic motion [7]. Their terms can be either computed analytically or represented as Fourier series.

Semi-analytical propagation is fast because it only requires a numerical integration of the slowly-varying component through long steps, together with a closed-form evaluation of the fast-dynamics component at the final epoch. In addition, the short-period expansions include coefficients that vary slowly, which allows being interpolated on a grid, similarly to the numerical integration of the mean elements. At the same time, semi-analytical propagation is also accurate, because it includes a wide range of perturbations [8].

Both Batch Least Squares and Kalman Filter processes estimate the DSST mean elements from the observation data [9]. The Extended Semi-analytical Kalman Filter (ESKF) employs analytical re-linearization on the observation grid [10].

It is worth noting that the step size in the case of the slow-dynamics component integration assumes that the mean element motion frequencies due to the zonal harmonics in the geopotential are slow relative to the frequencies due to the lunar perturbations. The integration step for a LEO orbit is usually around half a day, which corresponds to several revolutions. However, as the orbital period increases, the step size remains the same because the lunar-solar frequencies do not change, although the number of steps per revolution may be different due to the variation in the revolution duration.

The outline of this paper is as follows. First, the mean-element equations of motion are derived, and the perturbation models considered in DSST are described. Then, a brief introduction to DSST semi-analytical least square orbit determination is presented. After that, some numerical tests are discussed. Finally, we summarize this paper.

Mathematical Preliminaries

The theory underlying DSST makes use of non-canonical elements and is based on the Generalized Method of Averaging (GMA), an introduction to which can be found in Section 5.2.3 of Ref. 11, and Section 5.3 of Ref. 12. The GMA supports both the mean-element motion and the short-periodic motion, although in this section we will only focus on the mean-element equations of motion.

There are different forms of the Method of Averaging development for the cases of multiple small parameters and for time dependencies [13]. Both the Lagrange and Gauss forms of variation of parameters (VOP) are employed. The Lagrange form is used for the conservative terms: geopotential, lunar-solar point masses, and the solid Earth tides. This leads to the averaged disturbing potential. The Gauss form is used for the atmospheric drag and the solar radiation pressure (SRP). For drag and SRP, the integrals are evaluated by means of numerical quadrature. This approach facilitates the inclusion of complex atmosphere densities and spacecraft models in the DSST.

The short-periodic models are developed as a Fourier series in a rapidly varying phase angle; the phase angle is chosen to obtain closed forms for the Fourier series for the zonal and lunar-solar short-periodic motion [14]. The Fourier coefficients are functions of the mean equinoctial elements. The tesseral resonance and tesseral short-periodic models employ Jacobi polynomials and Hansen coefficients [15]. The detailed models emphasize the application of recursions. The calculations employ numerical interpolation techniques so that the detailed evaluations of the mean element rates and the Fourier coefficients are done only on large time grids.

A list of the force models for the mean-element and short-periodic motions is summarized in Table 1.

Table 1: Force model formulation for the Draper Semi-analytical Satellite Theory

Mean-element motion (Averaged VOP equations)	**Short-periodic motion** (Mean-to-osculating equations)
Recursive zonals in closed form and J_2^2 up to first order in eccentricity J_2^2via numerical quadrature [16]	Recursive zonals in closed form and J_2^2 up to first order in eccentricity
Recursive tesseral resonance (e^n, n>20) up to 50 x 50 geopotential	Recursive tesseral m-dailies in closed form Recursive tesseral linear combinations (e^n, n>20) Recursive J_2 secular / tesseral m-daily coupling in closed form
Recursive Solar-Lunar single averaged (time independent) in closed form Recursive Solar-Lunar double averaged	Recursive Solar-Lunar in closed form Recursive Solar-Lunar double averaged (weak time dependent) in closed form
Solid Earth tide in closed form	
Atmospheric drag via numerical quadrature J_2 – drag coupling terms via numerical quadrature	Atmospheric drag numerical computation
Solar radiation pressure via numerical quadrature	Solar radiation pressure numerical computation

The following physical perturbations will be considered as small parameters: both the zonal and the resonant tesseral harmonics in the non-spherical gravitational potential field, and the lunar-solar point mass factors. The resonant tesseral harmonics are those in which a commensurability between the periodic component of the perturbation, related to the central body rotation rate, and the mean motion of the satellite can be found, thus converting such periodicity into a steady variation. The slowly-varying resulting terms survive the averaging process of the GMA [17].

McClain [7] provides a derivation of the mean-element equations of motion for the case of two perturbing functions. The osculating equations of motion are given by

$$\begin{aligned}\frac{da_i}{dt} &= \varepsilon F_i(\boldsymbol{a}, l) + \nu G_i(\boldsymbol{a}, l) \qquad (i = 1, 2, \ldots, 5), \\ \frac{dl}{dt} &= n(\boldsymbol{a}) + \varepsilon F_6(\boldsymbol{a}, l) + \nu G_6(\boldsymbol{a}, l),\end{aligned} \tag{1}$$

where the quantities ε and ν are the small parameters and $\boldsymbol{a} = (a_1,\ldots,a_5)$. The a_i (i = 1,2,...,5) represent the equinoctial elements a, h, k, p, and q. The phase angle l is the mean longitude. The GMA involves the assumption of a transformation from the osculating mean elements,

$$\begin{aligned}a_i &= \bar{a}_i + \sum_{\substack{j=0 \\ (1\le j+k)}}^{N} \sum_{k=0}^{M(j)} \varepsilon^j \nu^k \psi_{i,j,k} + \mathcal{O}(\varepsilon^{N+1}), \\ l &= \bar{l} + \sum_{\substack{j=0 \\ (1\le j+k)}}^{N} \sum_{k=0}^{M(j)} \varepsilon^j \nu^k \psi_{6,j,k} + \mathcal{O}(\varepsilon^{N+1}),\end{aligned} \tag{2}$$

to the osculating elements, and an assumed form for the mean-element equations of motion,

$$\begin{aligned}\frac{d\bar{a}_i}{dt} &= \sum_{\substack{j=0 \\ (1\le j+k)}}^{N} \sum_{k=0}^{M(j)} \varepsilon^j \nu^k B_{i,j,k} + \mathcal{O}(\varepsilon^{N+1}), \\ \frac{d\bar{l}}{dt} &= n(\bar{a}_1) + \sum_{\substack{j=0 \\ (1\le j+k)}}^{N} \sum_{k=0}^{M(j)} \varepsilon^j \nu^k B_{6,j,k} + \mathcal{O}(\varepsilon^{N+1}).\end{aligned} \tag{3}$$

The functions $\psi_{i,j,k}$ depend on the mean elements $\bar{\boldsymbol{a}}, \bar{l}$ and are 2π periodic in the mean fast variable, $\bar{l}$. The functions $B_{i,j,k}$ only depend on the slowly varying elements $\bar{\boldsymbol{a}}$. The identity transformation is Eq. (2) and the assumed mean-element equations of motion are Eq. (3). For lunar-solar perturbations, the $\psi_{i,j,k}$ and the $B_{i,j,k}$ may also have dependencies on the lunar-solar ephemeris, not just the mean equinoctial elements.

At the conclusion of McClain's derivation, the mean-element equations of motion to first order reduce to

$$\begin{aligned}\frac{d\bar{a}_i}{dt} &= \varepsilon \langle F_i(\bar{\boldsymbol{a}}, \bar{l})\rangle_{\bar{l}} + \nu \langle G_i(\bar{\boldsymbol{a}}, \bar{l})\rangle_{\bar{l}} \qquad (i = 1, 2, \ldots, 5), \\ \frac{d\bar{l}}{dt} &= n(\bar{a}_1) + \varepsilon \langle F_6(\bar{\boldsymbol{a}}, \bar{l})\rangle_{\bar{l}} + \nu \langle G_6(\bar{\boldsymbol{a}}, \bar{l})\rangle_{\bar{l}}.\end{aligned} \tag{4}$$

Eq. (1) through Eq. (4) can be generalized to an arbitrary number of perturbing functions in a straight-forward manner. The mean-element equations of motion in the DSST software correspond to the generalization of Eq. (4).

Orbit Determination with DSST

A Batch Least Square (BLSQ) orbit determination (OD) method is applied extensively in most existing orbit determination systems for catalogue maintenance. The intention of this part of the work is to reduce the computational effort in this process; for this purpose, the DSST has been coupled with the BLSQ orbit determination method. A semi-analytical way of computing partial derivatives for a perturbed object which is compatible with the DSST was formulated by Green [9]. This section provides a brief insight into the selected part of Green's work.

For a given initial condition of a space object, with state X_{t0} associated with covariance P_{t0}, and for an available arc of observations, BLSQ provides the best estimate at the epoch state,

$$\hat{\mathbf{X}}_{t_0} = \mathbf{X}_{t_0} + \delta\mathbf{x}_0 \tag{5}$$

This is carried out in an iterative process by solving a Normal equation

$$\delta\mathbf{x}_0 = (A^T W A)^{-1} A^T W \mathbf{b} \tag{6}$$

where A is the partial derivative matrix, W is the weighting matrix and **b** represents the residual vector.

The derivation and preparation of the components of the Normal equation are described in [15]. The partial derivative matrix, A, is usually composed of the 'observation matrix: H' and the 'state transition matrix: Φ' (STM),

$$A = \frac{\partial\alpha(t)}{\partial\mathbf{X}(t)}\frac{\partial\mathbf{X}(t)}{\partial\mathbf{X}_{t_0}} = H_{t,t}\cdot\Phi_{t,0} \tag{7}$$

$\Phi_{t,0}$ can be computed by finite differencing method or by integrating the variational equations. $\Phi_{t,0}$ provides the slope and direction to the least square system towards convergence. To reach convergence (best estimate) in a smaller number of iterations, one has to include the major perturbing forces while establishing $\Phi_{t,0}$.

A runtime profiling of an OD system showed that the computation of the STM was one of the major resource consuming parts of that OD software. The profiling was carried out on a BLSQ program which used the numerical propagator, and the same is assumed to be true for other OD processes. The DSST-OD computes the STM in a semi-analytical fashion, thereby intending to reduce the computational load of the OD.

DSST employs propagation in the mean-element space, while the observations are in the osculating space. To map the mean space to the observed osculating space, the STM is further divided into two components, as follows

$$\Phi = \frac{\partial\mathbf{X}_t}{\partial\underline{\mathbf{c}}(t)}G \tag{8}$$

Here, $\mathbf{X}_t$ represents the osculating positions and velocities and $\underline{\mathbf{c}}(t)$ represents the osculating equinoctial elements at an arbitrary time, *t*. *G* represents the perturbed partial derivatives, which are expressed as

$$G = \left[\left\{\frac{\partial\underline{\mathbf{c}}(\mathbf{t})}{\partial\bar{\mathbf{c}}_0}\right\}\left\{\frac{\partial\underline{\mathbf{c}}(t)}{\partial\underline{\mathbf{p}}}\right\}\right] \tag{9}$$

with $\bar{\mathbf{c}}_0$ as the epoch mean equinoctial elements, and $\underline{\mathbf{p}}$ as the vector composed of the dynamical parameters in the least square estimation. For handling computation in a modular way, matrix *G* is further expanded as

$$G = [I + B_1][B_2 \mid B_3]] + [0 \mid B_4] \tag{10}$$

Matrices B_2 and B_3 are the partial derivatives of the mean elements at an arbitrary time with respect to the epoch time mean elements and the solve-for parameters, respectively. Matrices

B_1 and B_4 represent the short periodic portion of the semi-analytical partial derivatives, which are computed at the observation time intervals.

Matrices B_2 and B_3 are governed by linear differential equations, which are computed on the mean-element integration time grid. For the DSST, this is usually on the order of a half or one day step sizes.

Numerical results

With the aim of verifying the long-term propagation capabilities of DSST, we will present two cases that correspond to orbits with high eccentricities: a Molniya and a Super-GTO. Both cases are taken from [18]. The Molniya case is shown in Figure 1. This test case includes J_2-J_8 zonal harmonics, lunar and solar masses and solar radiation pressure, and was propagated using GTDS/DSST in J2000 coordinates. Only mean eccentricity and inclination are displayed. The initial orbit is 45800 km x 7460 km in apogee and perigee. The eccentricity is 0.72 at 63.4 degrees of inclination (critical). The periodic behavior in eccentricity and inclination indicates a period of approximately 8.33 years. Differences in along and cross track stay below 40 km and 10 meters, respectively, over a 50-year span, showing good agreement between NASA FDF and TRAMP files [19, 20, 21].

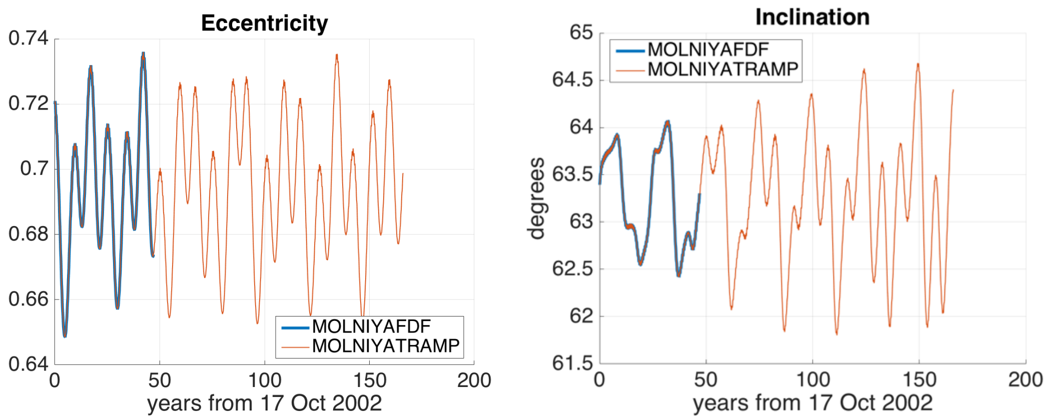

Fig. 1: Molniya mean eccentricity and inclination with J_2-J_8, L-S point masses, and SRP propagated in J2000 coordinates

The SuperGTO case is shown in Figure 2. This case includes J_2-J_8 zonal harmonics, J_2-squared using a quadrature model, lunar and solar point masses, and solar radiation pressure. This case was propagated using the GTDS/DSST in J2000 coordinates and, as the previous case, only mean eccentricity and inclination are displayed. The initial orbit is 96380 km x 6678 km in apogee and perigee. The periodic behaviors in eccentricity and inclination are similar, approximately 8 years. The initial eccentricity is 0.87 at 30 degrees of inclination. Differences between the propagations using NASA FDF and TRAMP files remain under 0.1 meters for the first 3 years where real A.1-UT1 data is available. The differences in radial, along and cross track stay below 10 km, respectively over a 14-year span. This shows good agreement between NASA FDF and TRAMP files.

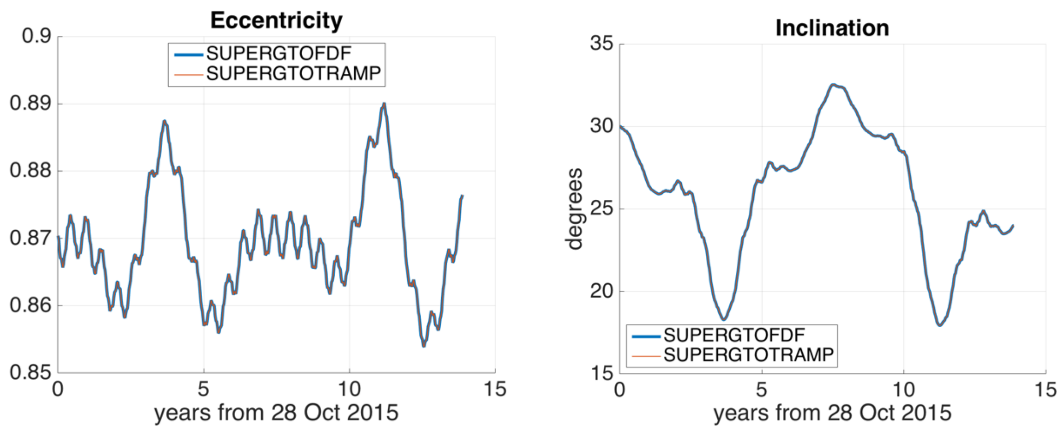

Fig. 2: SuperGTO *mean eccentricity and inclination with J2-J8,* J2-squared, *L-S point masses, and SRP propagated in J2000 coordinates*

Conclusions

This paper provides a short review of the Draper Semi-Analytical Satellite Theory (DSST) and some of its main features, such as the use of mean equinoctial elements as the state variables of the theory, or the interpolation system, which provide an efficient evaluation system of the mean elements and short-period terms.

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