Reachability Study for Spacecraft Maneuvering from a Distant Retrograde Orbit in the Earth-Moon System

Changxuan Wen [1] and Yang Gao [1]

[1] *Key Laboratory of Space Utilization, Technology and Engineering Center for Space Utilization, Chinese Academy of Sciences, Dengzhuang South Road No.9, Haidian District, Beijing, China*

Abstract

The low Earth orbit (LEO) reachability for a spacecraft maneuvering from a distant retrograde orbit (DRO) in the Earth-Moon system with a small velocity impulse is studied. The motivation behind this study is the plan of placing a DRO servicing platform which can fly by LEO targets at a low fuel cost. Such a mission is possible because the platform can maneuver from the DRO to the Moon with a small velocity increment. Then, the lunar gravity assist can be used to decelerate the platform, which enables it to reach LEO targets. Simulations show that a platform initially moving on a stable DRO can maneuver to the Moon from anywhere of the DRO and establish a lunar gravity assist with a velocity impulse as small as 50 m/s. The LEO reachability is analysed in terms of the reachability of the lowest altitude. By using the proposed maneuver strategy, reaching a LEO target requires a ΔV as small as 90 m/s.

Keywords: Distant retrograde orbit, impulsive orbit maneuver, reachability, gravity assist, Earth-Moon system

Introduction

Distant retrograde orbits (DROs) are a family of stable orbits around the smaller primary body in a restricted three-body system. The DRO in the Earth-Moon (EM) system has attracted increasing interests in recently years because of their unique characteristics, for example, to avoid the Earth's magnetic and radiation fields and to keep the distance between the spacecraft and the Earth below a certain predefined value [1]. Xu [2] proposed the idea of placing a transmitter on a DRO in the EM system and studied the low energy transfer to DRO.

Existing study shows that a space object can stably move on a DRO for hundreds of years [3]. Such a characteristic makes the DRO a promising storage orbit for space assets. NASA's Asteroid Redirect Robotic Mission (ARRM) plans to place a retrieved asteroid to a stable DRO in the EM system[4,5]. Bezrouk [6] studied the ballistic capture of an asteroid into the DRO.

Most of existing works have focused on how to transfer a satellite from the Earth to a DRO [1,2], or an asteroid from interplanetary space to the DRO in the EM system [4–6]. However, the inverse problem of transferring a spacecraft from a DRO to the LEO targets has been rarely studied. This study concentrates on the LEO reachability of a spacecraft initially moving on a stable DRO.

For a spacecraft moving on a DRO, it can maneuver from the DRO to the Moon with a small velocity increment. Then, the lunar gravity assist (LGA) can be used to decelerate the platform, which enables it to reach LEO targets. This means that, by using the LGA, a spacecraft can maneuver from a DRO to a low Earth orbit (LEO) at a low fuel cost. This motivates the plan of placing a servicing space platform on a DRO in the Earth-Moon (EM) system to service LEO targets. To support such a plan, a comprehensive analysis of the LEO reachability for a spacecraft maneuvering from a DRO with a small available velocity increment (due to limited fuel storage) is necessary.

This paper is organized as follow. First, a maneuver strategy utilizing the LGA is proposed to transfer from a DRO to reach LEO targets. Then, the Moon's reachability is studied because the proposed the maneuver strategy requires the LGA. Finally, the LEO reachability is considered by solving the radius of perigee. The current study only considers the planar case in the EM plane. Besides, the following three practical mission constraints are imposed on the transfer orbit:

(1) The time of flight, t_{tof}, is constrained to be no more than two orbit periods of the Moon, roughly about two months.

(2) To guarantee the safety of the LGA, the altitude of the orbit with respect to the Moon's surface should be larger than 100 km. When the altitude is smaller than 100 km, the spacecraft is thought to have been collided into the Moon.

(3) The transfer only uses one LGA. This means that the spacecraft is not allowed to enter the Moon's sphere of influence (SOI) for the second time. When the spacecraft left the Moon's SOI for the first time, it is not allowed to enter the Moon's SOI again.

Maneuver Strategy from a DRO to LEO

By imposing a small velocity impulse in arbitrary direction on a spacecraft moving on a stable DRO, the spacecraft will leave the original DRO orbit. The new transfer orbit after the maneuver can either approach to or depart from the Moon. For those transfer orbit approaching to the Moon and entering the Moon's SOI, they will be significantly influenced by the Moon's gravity to alter the flight trajectory, which is generally known as the LGA.

Only by utilizing the LGA that a spacecraft can be significantly decelerated to reach a LEO target. Such a transfer profile can be summarized as "approaching to and decelerated by the Moon", which is shown in Fig. 1. This transfer profile consists of three essential phases: (1) the spacecraft deviates from the DRO by imposing a small impulse $\Delta\mathbf{v}$ and coasts to the Moon, (2) it travels through the Moon's SOI to finish the LGA, and (3) it departs from the Moon's SOI and transfer to intercept the LEO targets after the LGA.

In this study, we focus on the LEO reachability for a spacecraft maneuvering from a stable DRO and following the given transfer profile.

Suppose that the spacecraft is initially moving on a stable DRO with the periodic around the Moon in the rotating EM system being τ. Let $\mathbf{x}_0$ denote the state vector of the spacecraft (in the EM rotating coordinate system) when it traverses the EM line with the velocity pointing in the positive direction, thus,

$$\mathbf{x}_0 = \begin{bmatrix} \mathbf{r}_0^T & \mathbf{v}_0^T \end{bmatrix}^T = \begin{bmatrix} x_0 & 0 & 0 & \dot{y}_0 \end{bmatrix}^T, \dot{y}_0 > 0 \tag{1}$$

For a stable DRO outside the Moon's SOI, the value of x_0 should be chosen between 0.73 and 0.81 [3]. In this research we consider a family of stable DRO with $x_0 \in X_0$, where

$$X_0 = \{0.73, 0.74, 0.75, 0.76, 0.77, 0.78, 0.79, 0.80, 0.81\} \tag{2}$$

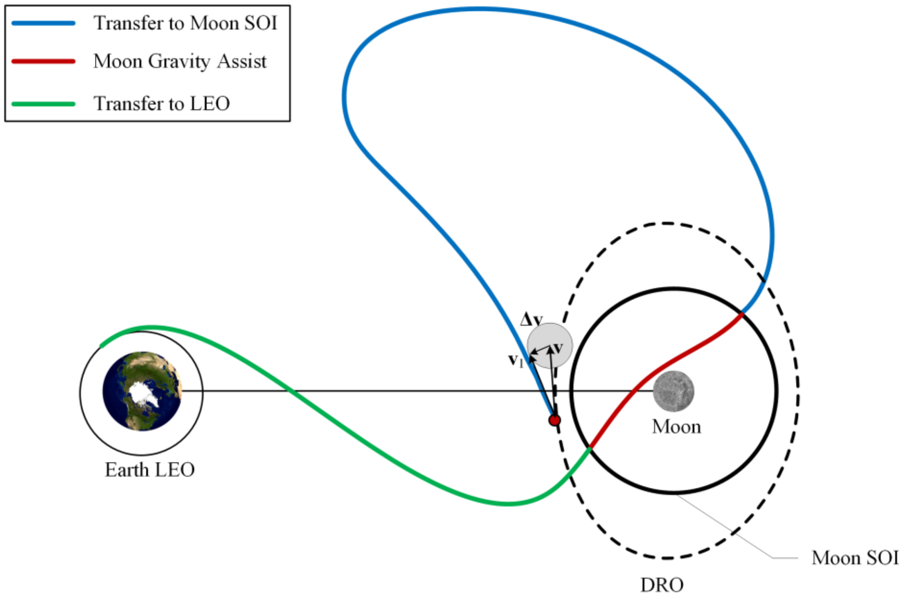

Fig. 1: Maneuver strategy from DRO to LEO in the EM rotating coordinate system

The maneuver velocity impulse $\Delta\mathbf{v}$ is imposed right before the spacecraft leave the DRO. The maneuver position on the DRO and the velocity impulse direction are arbitrary. However, the magnitude of $\Delta\mathbf{v}$ is constrained to the maximum available Δv. The maneuver position, $\mathbf{r}$, on the DRO is a control variable. Let $\mathbf{x}=\begin{bmatrix}\mathbf{r}^T & \mathbf{v}^T\end{bmatrix}^T$ be the state vector of the spacecraft on the DRO before the maneuver. The corresponding state vector of the DRO when traversing the EM line is $\mathbf{x}_0$. Obviously, $\mathbf{x}$ can be propagated from $\mathbf{x}_0$ by a flight time interval of Δt. Thus, the maneuver position $\mathbf{r}$ on the DRO depends on Δt. When Δt traverses the interval $[0,\tau)$, the maneuver position covers the entire DRO. Now we define a dimensionless control variable η as

$$\eta=\frac{\Delta t}{\tau}\in[0,1) \tag{3}$$

which decides the maneuver position on the given DRO.

The direction of the maneuver velocity impulse $\Delta\mathbf{v}$ can be arbitrary. Let α denote the angle between $\Delta\mathbf{v}$ and $\mathbf{v}$. Thus, $\alpha\in[0,2\pi)$ is another control variable that decides the direction of $\Delta\mathbf{v}$.

Based on the definitions above, the two control variables for the Maneuver strategy from DRO to LEO is (η,α). The permissible control set P satisfies

$$P=\left\{(\eta,\alpha)\middle|\eta\in[0,1),\alpha\in[0,2\pi)\right\} \tag{4}$$

The state right after the impulsive maneuver is denoted as $\mathbf{x}_1=\begin{bmatrix}\mathbf{r}_1^T & \mathbf{v}_1^T\end{bmatrix}^T$, which can be obtained as

$$\begin{cases}\mathbf{r}_1(\eta,\alpha)=\mathbf{r}(\eta)\\ \mathbf{v}_1(\eta,\alpha)=\mathbf{v}(\eta)+\dfrac{\Delta v}{\|\mathbf{v}(\eta)\|}\mathbf{R}_z(\alpha)\mathbf{v}(\eta)\end{cases},\quad (\eta,\alpha)\in P \tag{5}$$

where $\mathbf{R}_z(\alpha)$ is a rotation matrix,

$$\mathbf{R}_z(\alpha)=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix} \tag{6}$$

The maneuver strategy from a DRO is completely defined. Next, both the Moon's reachability and the LEO reachability will be studied.

Moon's Reachability

For the proposed maneuver strategy in the previous section, the LGA is possible only for transfers entering the Moon's SOI. Thus, the Moon reachability is firstly analysed.

For those transfer trajectories approaching to the Moon, a minimum flight altitude $h_{\min}$ is set to avoid of impacting the Moon. Generally, the flight trajectories after the impulsive orbit control from the DRO orbit fall into three categories:

(1) LGA trajectories. Those are the flight trajectories that enters the Moon's SOI, and meanwhile altitude of their perilune is larger than $h_{\min}$.

(2) Moon collision trajectories. Those are the flight trajectories that enters the Moon's SOI, and meanwhile altitude of their perilune is less than $h_{\min}$. These flight trajectories are considered as impacting the Moon.

(3) No LGA trajectories. Those are the flight trajectories that do not enter the Moon's SOI on the whole time duration of integral.

When the control variable (η,α) traverses the entire control set P, a plot of the distribution of the above three categories can be obtained. Also, the proportion of the LGA category, denoted by the symbol λ, can also be evaluated. It is evident that the larger the λ is, the easier the LGA can be accomplished. Thus, λ can be treated as a performance index to characterize the Moon reachability of a maneuver from a DRO.

We consider an example to illustrate the Moon's reachability problem. In this study, all variables are normalized. For the EM system, a length unit 1DU represents 3.84748×10^{8} m, and a velocity unit 1VU represents 1.02408×10^{3} m/s. All the parameters (normalized) in this DRO maneuver scenario are set as

$$x_0 = 0.76,\ \Delta v = 0.1 \tag{7}$$

$$h_{\min} = 100\ \text{km} = 2.5991\times10^{-4}\ \text{DU} \tag{8}$$

With the setting above, the simulated Moon's reachability is given in Fig. 2, where the color green, red, and blue represent the transfer trajectories of "No LGA", "Moon collision", and "LGA" categories, respectively. The proportion of the LGA category, which accounts for the proportion of the color blue in the figure, is $\lambda = 59.6\%$.

It can also be found that, at the neighbourhood of $\alpha = \pi$, most transfer trajectories can enter the Moon SOI, resulting in either LGA or Moon collision. This is because when $\alpha = \pi$, the velocity impulse $\Delta\mathbf{v}$ is imposed in the anti-velocity direction so that the spacecraft is decelerated and thus moves towards the Moon. Meanwhile, at the neighbourhood of $\alpha = 0$ and 2π, most transfer trajectories do not enter the Moon's SOI, as shown by the green area in Fig. 2.

The three subfigures on the right of Fig. 2 shows three representatives corresponding to the three transfer trajectory categories. It is observed that the green flying trajectory does not enter the Moon SOI, which cannot accomplish an LGA. The red flight trajectory flies too close to the Moon so that it collides into the Moon. The blue flight trajectory successfully enters the Moon's SOI and finally accomplishes an LGA. Fig. 2 also shows that for $\forall\eta \in [0,1)$, there is always blue area. This means that the spacecraft can enter the Moon's SOI by imposing the velocity impulse at any position on the DRO. However, for those transfer trajectories enters the Moon's SOI, part of them collide into the Moon and the rest of them successfully accomplish the LGA.

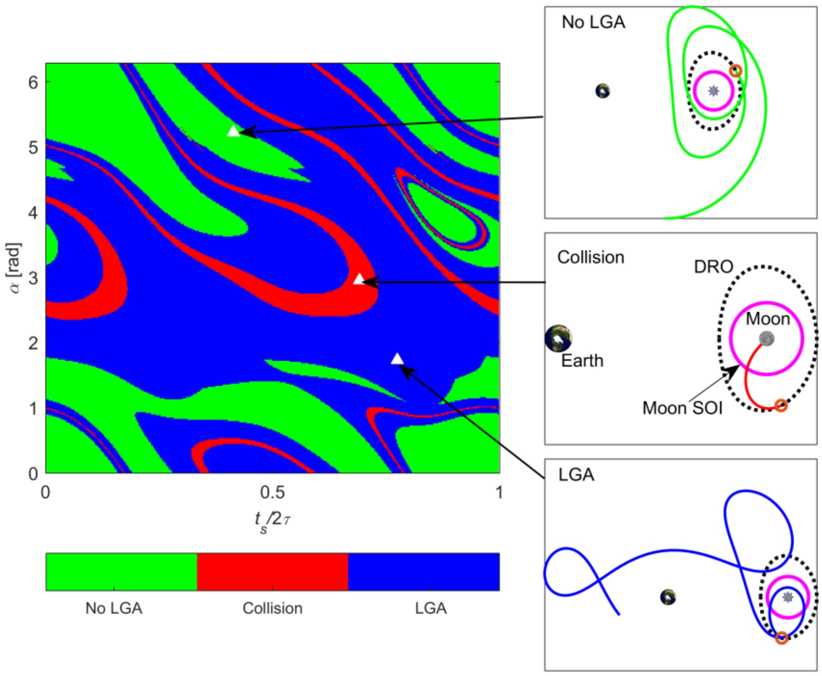

Fig. 2: Moon's reachability analysis ($x_0 = 0.76$, $\Delta v = 0.1$)

We now consider the cases for different initial stable DROs. For $x_0 \in X_0$ and $\Delta v \in [0.04, 0.18]$, the results for the performance index λ are shown in Fig. 3.

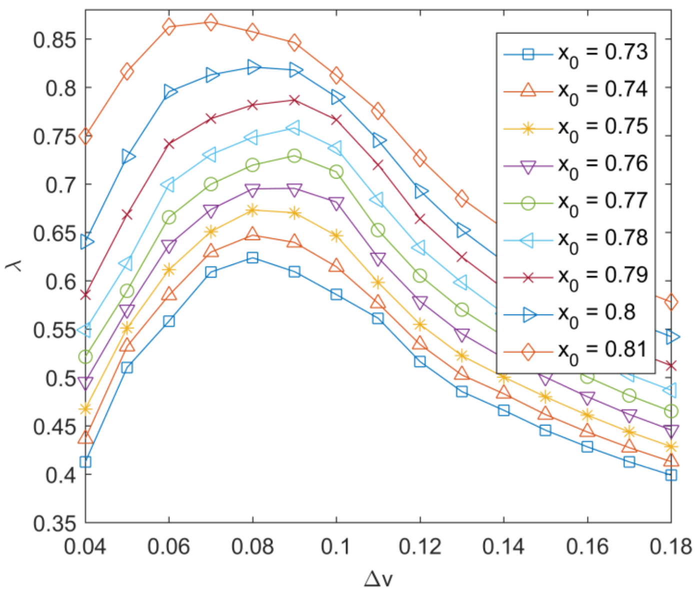

Fig. 3: Moon's reachability for different maneuver scenarios

Fig. 3 shows that, for a given Δv, λ increases with x_0. This is because for a DRO with larger x_0, it is closer to the Moon and therefore it is easier the reach the Moon. However, for a given DRO (x_0), λ firstly increases and then decreases with Δv; the maximum of λ occurs when $\Delta v \approx 0.08$. This phenomenon is against our instinct that the Moon can be reached easier when the velocity impulse is larger. This can be accounted for by the fact the transfer orbit can escape the Moon's SOI when Δv is large enough. Since the Moon's gravity cannot pull the spacecraft into its SOI, the Moon's reachability is getting worse when Δv is larger than a critical point around 0.08. Such a result reveals that increasing of Δv does not essentially improve the Moon's reachability. In fact, our simulation shows that when Δv is between 0.7 (72 m/s) to 0.9 (92 m/s) for different DRO, the Moon's reachability reaches its peak.

Low Earth Orbit Reachability

For a DRO maneuver with x_0 and Δv, the transfer orbit can be generated by providing a pair of control variable (η,α) and the flight time constraint $t_{tof} \le 4\pi$. Thus, a perigee on the transfer orbit is readily to evaluate, which is denoted as $r^p(\eta,\alpha)$

$$r^p(\eta,\alpha) = \inf\left(\left\|\mathbf{r}(t;\eta,\alpha) - \mathbf{r}_E\right\|\right), t \in [0, 4\pi) \tag{9}$$

where $\mathbf{r}_E = [-\mu \quad 0]^T$ is the location of the Earth center. Define the global minimum of $r^p(\eta,\alpha)$ as

$$r^p_{\min} = \inf\left[r^p(\eta,\alpha)\right], (\eta,\alpha) \in P \tag{10}$$

Thus, $r^p_{\min}$ can be used as a performance index for LEO reachability of the transfer orbit. Let the corresponding control variables being (η^*,α^*), satisfying

$$r^p(\eta^*,\alpha^*) = r^p_{\min} \tag{11}$$

where (η^*,α^*) is the optimal control variables.

Given an initial DRO with $x_0 = 0.77$ and a maneuver velocity imuplse of $\Delta v = 0.09$, by traversing (η,α) the LEO reachability of the transfer orbit is presented in the left subfigure of Fig. 4. The color bar shows the magnitude of $r^p(\eta,\alpha)$, with the blue direction of the color bar representing the lower value (closer to the Earth) and the yellow direction representing the higher value (far away from the Earth). The closed white curve is the contour of $r^p(\eta,\alpha) = r_{geo}$, where $r_{geo} = 0.1109$ is the radius of the geostationary orbit (GEO). The dark blue area enclosed by the GEO contour is where the transfer orbit can reach earth orbit lower than the GEO belt.

The lowest transfer orbit is $r^p_{\min} = 0.09043$, and the corresponding optimal control variables are

$$\eta^* = 0.5083, \ \alpha^* = 6.2657$$

The right subfigures of Fig. 4 show the optimal transfer orbit in both the rotating and the Earth inertial frame.

To further analyze the LEO reachability, we consider different initial DROs $x_0 \in X_0$ and different Δv. For $\Delta v \in [0.05, 0.18]$, the history of $r^p_{\min}$ is shown in Fig. 5. It can be seen that for any given DRO, $r^p_{\min}$ decreases with Δv. According to the slop of the $r^p_{\min}$ line, it can be divided into three segments: slowly decreasing segment with small Δv, fast decreasing segment, and slowly decreasing segment with large Δv. For different initial DROs, the start of the fast decreasing segment varies between 0.06 and 0.08, and the end of this segment is around 0.09.

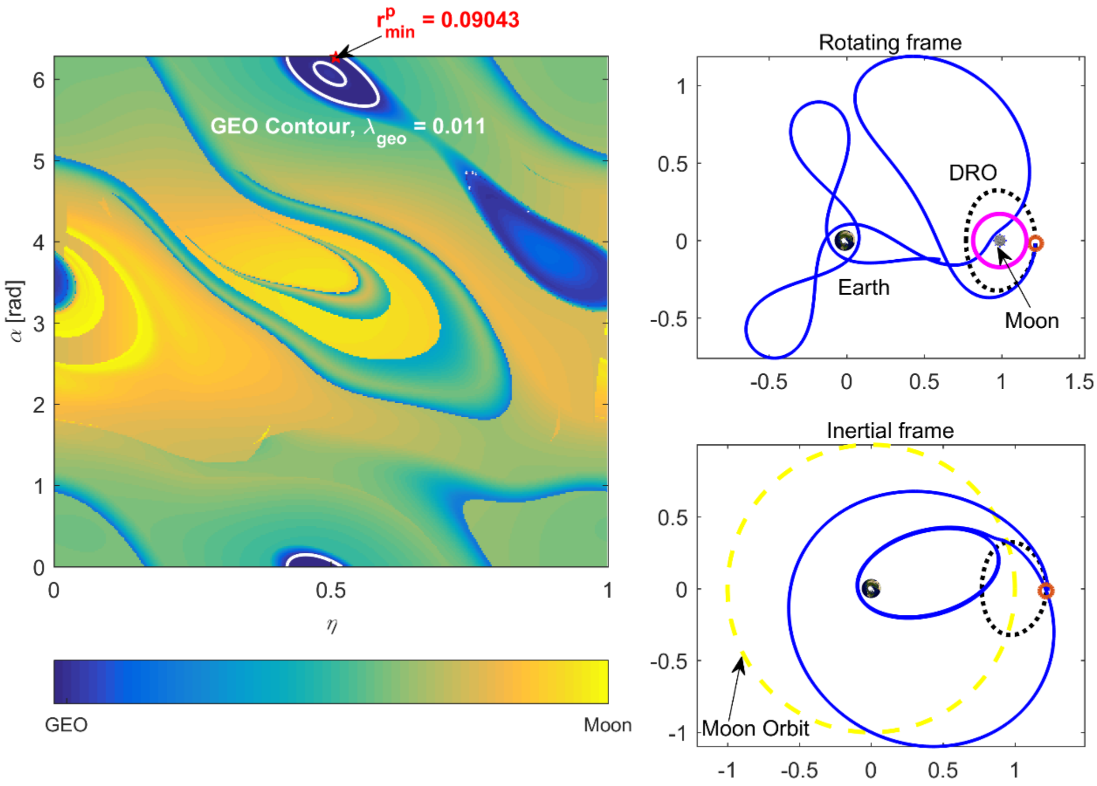

Fig. 4: Low Earth orbit reachability analyse for $x_0 = 0.76, \Delta v = 0.09$

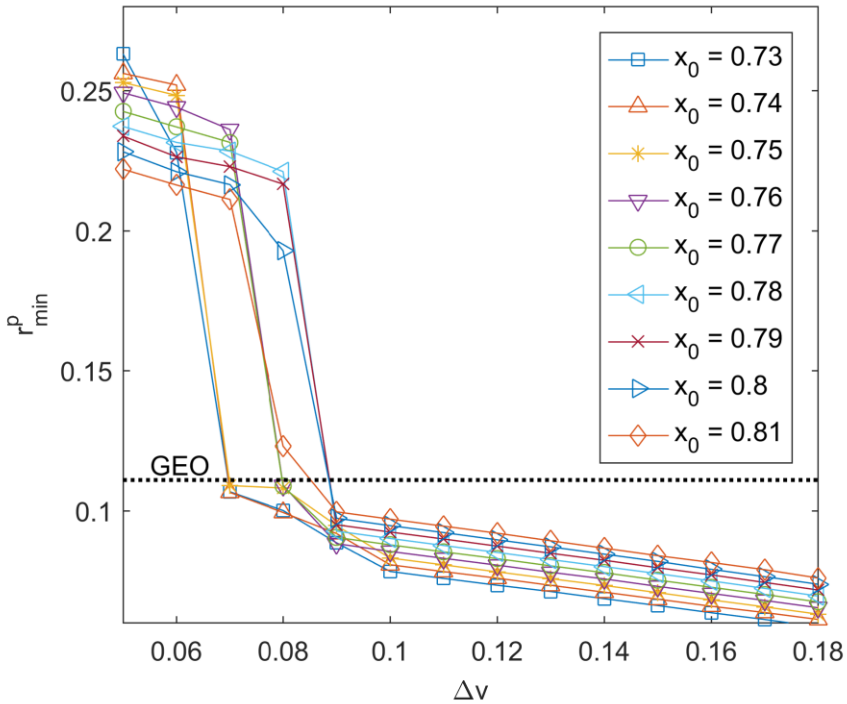

Fig. 5: Low Earth orbit reachability analyse for different scenarios

The numerical simulations have shown that, for LEO reachability, it is favoured to provide a Δv between 0.09 (~90 m/s) and 0.1 (~100 m/s) to guarantee the reachability and the efficiency simultaneously.

This simulation verifies that the LEO targets can be reached by a spacecraft maneuvering from a stable DRO in the EM system at a low fuel cost, with the required velocity impulse as small as 90 m/s.

Conclusion

This study proposed a maneuver strategy to reach the LEO space assets for a spacecraft initially moving on a stable DRO in the EM system. Simulations show that the a DRO platform can maneuver to the Moon from anywhere of the DRO and establish an LGA with a velocity impulse as small as 50 m/s. The LEO targets can be reached by using the proposed maneuver strategy when the velocity impulse is increased to about 90 m/s, which is a very low fuel cost. Results of this paper support the mission of placing a servicing space platform on a DRO in the EM system.

References

[1] J. Demeyer, P. Gurfil, Transfer to Distant Retrograde Orbits Using Manifold Theory, J. Guid. Control. Dyn. 30 (2007) 1261–1267. doi:10.2514/1.24960.

[2] X. Ming, X. Shijie, Exploration of distant retrograde orbits around Moon, Acta Astronaut. 65 (2009) 853–860. doi:10.1016/J.ACTAASTRO.2009.03.026.

[3] C.J. Bezrouk, J. Parker, Long Duration Stability of Distant Retrograde Orbits, in: AIAA/AAS Astrodyn. Spec. Conf., American Institute of Aeronautics and Astronautics, 2014. doi:doi:10.2514/6.2014-4424.

[4] D.D. Mazanek, R.G. Merrill, J.R. Brophy, R.P. Mueller, Asteroid Redirect Mission concept: A bold approach for utilizing space resources, Acta Astronaut. 117 (2015) 163–171. doi:10.1016/J.ACTAASTRO.2015.06.018.

[5] N. Strange, D. Landau, T. McElrath, G. Lantoine, T. Lam, Overview of mission design for NASA Asteroid Redirect Robotic Mission concept, in: 33rd Int. Electr. Propuls. Conf., Pasadena, CA : Jet Propulsion Laboratory, National Aeronautics and Space Administration, 2013, 2013. https://trs.jpl.nasa.gov/handle/2014/44361 (accessed January 12, 2018).

[6] C. Bezrouk, J.S. Parker, Ballistic Capture into Distant Retrograde Orbits from Interplanetary Space, in: Proc. AAS/AIAA Sp. Flight Mech. Meet. Pap. AAS, 2015: pp. 15–302.