An Integrity Assessment Method of Angles-Only Navigation Solutions in Non-Cooperative Rendezvous Operations

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***Abstract* – In non-cooperative rendezvous, Angles-Only Navigation (AON), which estimates the relative orbit by utilizing the history of LOS angles and the relative motion model, is a crucial navigation method at the middle and far ranges. A significant challenge AON faces is its limited observability, as the LOS angles lack range information for estimating relative trajectories. Consequently, even if the relative trajectory estimation seems to converge as a numerical solution, it may result far from the true trajectory. This study focuses on the "residuals curve" proposed by J.S. Ardaens et al. (2019) in the IROD problem. We introduce a method for evaluating the confidence of solutions obtained by AON while the true values are unknown in actual operations.**

I. INTRODUCTION

The need for non-cooperative rendezvous, including repairing satellite and debris removal, is expanding. In terms of debris removal missions, JAXA has launched the Commercial Removal of Debris Demonstration (CRD2) project to research removing large space debris [1]. Active Debris Removal (ADR) of large debris, which can be a source of small debris, effectively improves the space environment. Malfunctioning satellites and space debris are categorized as non-cooperative targets that do not have functions to support navigation, such as GNSS receivers or reflectors. Fig. 1 represents the navigation systems that our research assumed [2]. Firstly, the target trajectory is determined by radar observations from grand stations, and the navigation system switches to relative navigation systems when the optical camera starts to detect a target. At this point, the target is imaged as a tiny dot because the relative range is extensive. Therefore, only Line of Sight (LOS) angles can be extracted as meaningful data from measurements. Angles-Only Navigation (AON) is a navigation method used to estimate the relative state vector by utilizing the history of LOS angles and the relative orbit motion model.

In non-cooperative rendezvous, AON is a crucial navigation method at medium to far ranges. A significant challenge AON faces is its limited observability, as the LOS angles lack range information for estimating relative trajectories. Consequently, even if the relative trajectory estimation seems to converge as a numerical solution, it may result far from the true trajectory. In non-cooperative rendezvous operations, it becomes a critical issue to consciously prevent cases where the observability of AON is insufficient and to assess the integrity of its estimated trajectory.

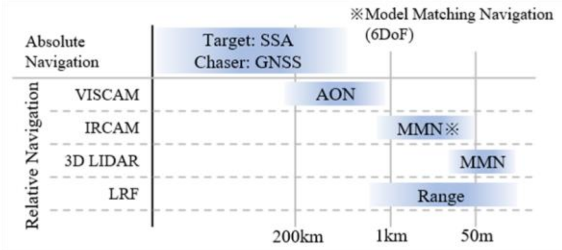


Fig. 1. Navigation sensor systems

In previous research on AON, the Initial Relative Orbit Determination (IROD) problem, which calculates the initial values used in AON estimation, has been actively discussed. We focus on the "residuals curve" proposed in the IROD problem. In this paper, the relationships between the steepness of the residuals curve and AON accuracy are investigated through Monte Carlo simulations to assess whether the integrity of AON can be appropriately evaluated using this curve.

II. ANGLES-ONLY NAVIGATION

AON, formally used for Space shuttle missions, was recently demonstrated in ARGON[3] and AVANTI[4], [5], which are the DLR's rendezvous missions. It has become a critical navigation method for non-cooperative rendezvous.

AON is the problem described by the following equations, a matching of target LOS vectors, which are estimated from the state $\boldsymbol{x}$ and relative motion model, with measurements $\boldsymbol{u}$.

$$\boldsymbol{u}(t_k) \times \boldsymbol{C}(t_k)\boldsymbol{\Phi}(t_k, t_0)\boldsymbol{x}(t_o) = 0 \quad (1)$$

$$\boldsymbol{x}^T = (a\delta a \quad a\delta e_x \quad a\delta e_y \quad a\delta i_x \quad a\delta i_y \quad a\delta u) \quad (2)$$

In the above equations, $\boldsymbol{x}$ are the relative orbital elements (ROEs); $\boldsymbol{C}$ is the measurement matrix; $\boldsymbol{\Phi}$ is the state transition matrix; and $\boldsymbol{u}$ are the LOS vectors in Fig. 2. The relative motion model in (1) has a linearized error, and there is a noise in measurements of $\boldsymbol{u}$. Therefore, the orbit determination problem of

AON is linearized around $\boldsymbol{x}^{apr}$ and becomes a problem of estimating $\boldsymbol{x}^{lsq}$, which minimizes the residuals $|\Delta\boldsymbol{y}|^2$.

$$\text{find } \boldsymbol{x}^{lsq} \text{ minimizing } |\Delta\boldsymbol{y}|^2 \tag{3}$$

where;

$$\Delta\boldsymbol{y} = \boldsymbol{H}\Delta\boldsymbol{x},\ \Delta\boldsymbol{y}^T = (\Delta\boldsymbol{y}_1^T, \dots, \Delta\boldsymbol{y}_k^T), \\ \Delta\boldsymbol{y}_k = \boldsymbol{\zeta}_k - \boldsymbol{y}_k,\ \Delta\boldsymbol{x} = \boldsymbol{x}^{lsq} - \boldsymbol{x}^{apr} \tag{4}$$

$\boldsymbol{y}$ and $\boldsymbol{\zeta}$ are the azimuth ψ and elevation η angles in figure X; $\boldsymbol{H}$ is observability matrix. This study uses the following weighted least-square orbit determination to solve (3).

$$\boldsymbol{x}_{j+1}^{lsq} = \boldsymbol{x}_j^{lsq} + \left(\boldsymbol{\Lambda}^{apr} + \boldsymbol{H}_j^T \boldsymbol{W} \boldsymbol{H}_j\right)^{-1}\left[\boldsymbol{\Lambda}^{apr}\left(\boldsymbol{x}_j^{lsq} - \boldsymbol{x}^{apr}\right) + \boldsymbol{H}_j^T \boldsymbol{W} \Delta\boldsymbol{y}\right] \tag{5}$$

$\boldsymbol{\Lambda}$ is information matrix; $\boldsymbol{W}$ is weighted matrix determined from measurements noise. Equation (5) starts from initial values $\boldsymbol{x}_0^{lsq} = \boldsymbol{x}^{apr}$ and iterates until convergence.

The linearized equation of (1) approximates the AON problem well. In addition to $\boldsymbol{x}$, $\mu\boldsymbol{x}$ can also be a solution to (1). Therefore, the challenge of AON lies in its weak observability, which is caused by the presence of numerous similar solutions. Considering the following factors enables enhancement of the observability:

- Changing relative orbit by maneuvers.
- The difference between the curvilinear (CVL) and Radial-Tangential-Normal (RTN) coordinates.

Under the presence of maneuvers $\boldsymbol{v}$, the future states are described as follows: $\boldsymbol{x}(t_k) = \boldsymbol{\Phi}(t_k, t_0)\boldsymbol{x}(t_o) + \boldsymbol{v}$, and the solution of (1) can be determined as unique. Compared to the RTN coordinate, the CVL coordinate is a cylindrical coordinate frame whose Tangential axis follows a circular orbit. Properly describing the difference stimulates observability. However, as shown in Fig. 3, the two coordinates become almost identical systems when the relative distance is closer. As a result, maneuvering is the only way to ensure observability at close range. One of the challenges of rendezvous design using AON is ensuring observability within the limitation of $\Delta\boldsymbol{v}$.

Fig. 4 shows an example result where the observability of AON is insufficient. Simulation conditions are the same as middle-range simulations in Section IV. The horizontal axis means the iteration number of (5), and the vertical axis is the estimation results of $a\delta u$ and its covariance (1σ). The figure confirms that AON converges with each iteration as a numerical solution. However, the converged result is significantly different from the true value, which is unexpected based on the estimated covariance. When the observability is too weak like this case, it is difficult to assess the validity of the AON solutions from the estimated state and covariance. It is needed to consciously prevent cases

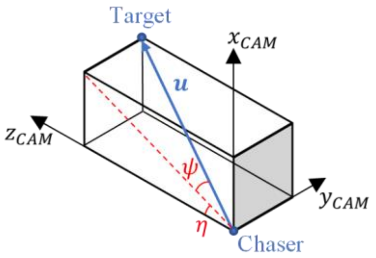


Fig. 2. Definition of measurements

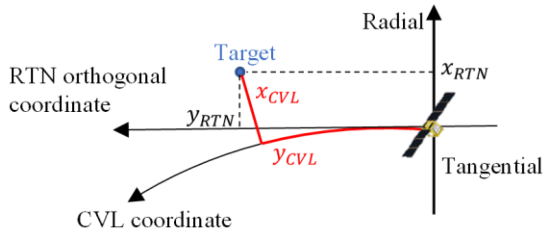


Fig. 3. CVL and RTN coordinates

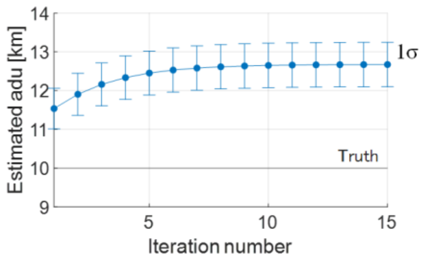


Fig. 4. AON with a lack of observability

where the observability of AON is insufficient and to assess the integrity of its estimated trajectory.

III. Integrity Assessment by Residual Curve

A. *Previous Research of J.S. Ardaens et al.*

In former research, the Initial Relative Orbit Determination (IROD) problem, which estimates the initial values used in the AON, has been actively discussed. This study focuses on the "residuals curve" proposed by J.S. Ardaens et al. (2019) [6] in the IROD problem.

As shown in Fig. 5, ROEs in (2) can be divided as follows: elements other than $a\delta u$ determine the shape of the relative orbit, while $a\delta u$ works as a scale factor which selects the one from the numerous similar trajectories.

Since the measurements of AON consist of LOS angles, estimating the shape of the relative trajectory in the radial–normal plane is relatively easy. The primary challenge, however, lies in estimating $a\delta u$, which selects the relative orbit, minimizing the residuals from similar orbits.

The residual curve highlights this characteristic of the AON. Firstly, to plot the curve, the shape of the relative trajectory is estimated from measurements, and a set of similar trajectories is obtained by setting arbitrary values for $a\delta u$. Then, observation residuals are calculated and plotted for each trajectory, resulting in the residual curve in Fig. 6. If the trajectory is close to the truth, the residuals should be small. Then, the relative orbits corresponding to the minimum residuals are selected as initial state $\boldsymbol{x}^{apr}$, and the weighted matrix $\boldsymbol{W}$ is also updated based on its residuals.

B. Proposed method

Previous research has suggested that the steepness of the curve is influenced by factors such as the amount of measurement noise, the presence of maneuvers, and the length of the orbit determination arc. We focused on this characteristic and investigated whether the observability of AON itself could be determined by the steepness of the residual curve.

As previously mentioned, AON fundamentally has weak observability in $a\delta u$, which operates as a scale factor for the similar expansion of relative orbital shape. However, to solve AON, it is essential to have sufficient observability, even if it is weak. AON rendezvous should be designed to ensure there is enough information in the measurements and the relative motion model to uniquely determine $a\delta u$. If not, then the trajectory lacks observability, and you are working on the AON problem that cannot be properly solved.

The problem the authors see here is that even for AON problems that are weakly observable and not properly solvable, the numerical solutions, such as by least-squares methods, may converge to $a\delta u$ far from the true value. In the actual operation of AON rendezvous, there is no correct value to compare, so there is a risk of using a greatly mistaken $a\delta u$ for maneuver planning.

If the estimated trajectory is closer to the true trajectory, the residual will be smaller. Therefore, by computing residuals from a trajectory estimation result with a fixed $a\delta u$ and executing this process for multiple values of $a\delta u$ while plotting the residuals against $a\delta u$, the resulting residual curve is expected to reach a minimum when $a\delta u$ equals the true value. This is because the true $a\delta u$ should generate a trajectory that is closer to the correct trajectory than the incorrect $a\delta u$. Conversely, if the residuals are not minimized in the true value of $a\delta u$, it should mean that there is not enough information to uniquely determine $a\delta u$, and designed AON rendezvous is inappropriate.

Therefore, in actual operation, not only checking the estimated state and covariance of AON, but also plotting and evaluating a residuals curve can reduce the risk of using the greatly mistaken $a\delta u$ mentioned above for maneuver planning. If the residuals curve is flat or if the local minimum of the curve is far from the estimated state, then we can notice that there is a problem with the AON observability. In this case, various factors that weaken observability, such as measurement noise, amount of measurements, modelling errors and amount of maneuvers, etc., need to be improved. It is important for the operator to be able to notice them.

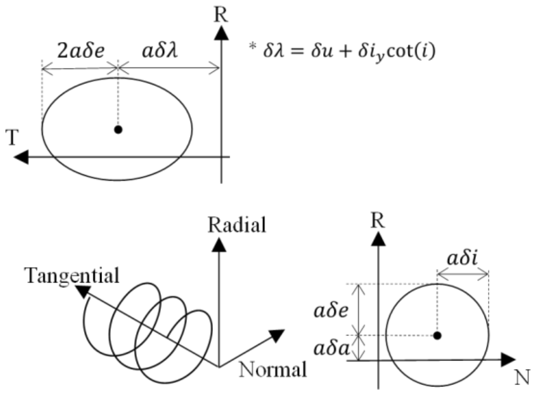


Fig. 5. Relative orbital elements

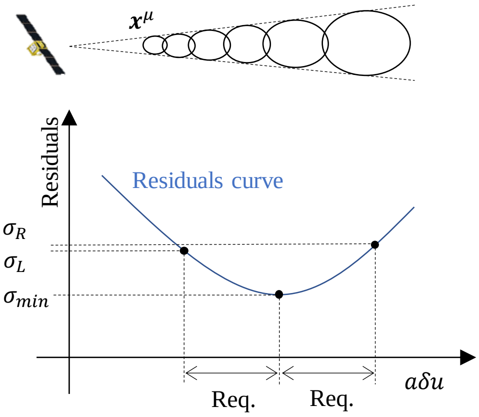


Fig. 6. Residual curve

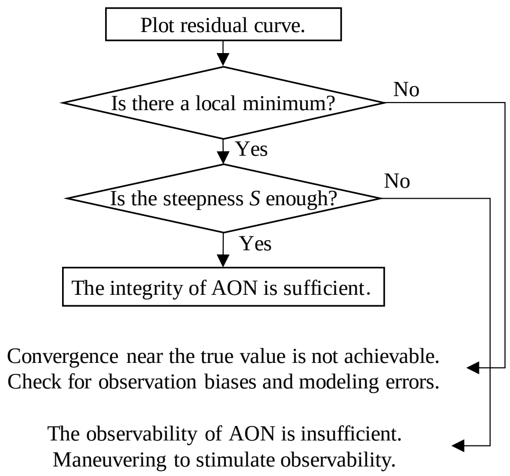


Fig. 7. Integrity assessment flow

Fig. 7 represents our proposed integrity assessment method. Initially, a residual curve is plotted for a sufficiently broad range of $a\delta u$ compared to the assumed accuracy of AON. If no local minimum exists, it is assumed that the results of AON are significantly biased due to observation biases or modeling errors. If a local minimum exists, the steepness of the residual curve

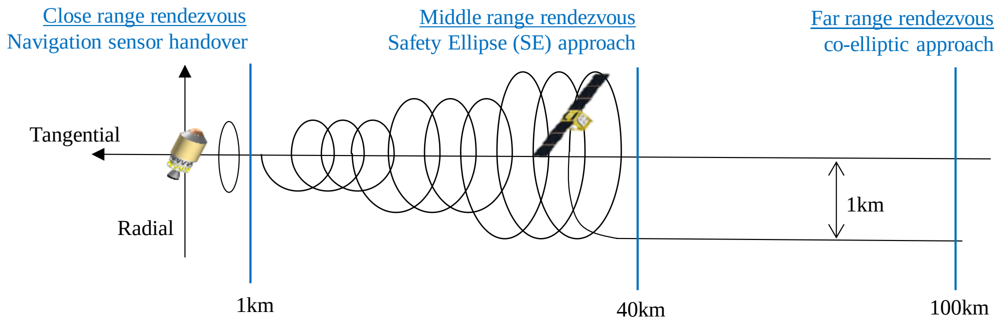


Fig. 8. Simulation scenario

is focused. This research defines the steepness as follows.

$$S = \left\{ \frac{(\sigma_L + \sigma_R)}{2\sigma_{min}} - 1 \right\} \cdot 100\ [\%] \qquad (6)$$

Fig. 6 explains the steepness S and the average increment of residuals, σ_L and σ_R. If the steepness is large, there would be sufficient observability for the required accuracy, and the relative orbit is estimated updating the initial states $\boldsymbol{x}^{apr}$ and weighted matrix $\boldsymbol{W}$. While if the steepness is small, larger maneuvers are necessary to enhance observability.

IV. NUMERICAL ANALYSIS

Monte Carlo simulations are conducted to assess whether the integrity of AON can be appropriately evaluated using the residual curve. Fig. 8 shows our simulation scenarios. In the simulations, the target becomes detectable from 100km, and AON is used until the distance is 1km. We prepared three rendezvous scenarios depending on the relative distance.

Simulation conditions are shown in Table 1. Atmospheric density errors are set to ±15% using a uniform distribution model. The visibility of the target is modeled based on BRDF measurements, and the camera is blinded when the sun is within 60 degrees of one side of the optical axis. To plot the residuals curve, the horizontal range is set to ±10% against the $a\delta u(t_o)$, and relative trajectories $\boldsymbol{x}^{\mu}$ are generated in increments of 1% of the horizontal range. In this paper, the required accuracy of AON is set to ±5% of $a\delta u(t_o)$.

In each scenarios, we conducted 30 Monte Carlo simulations. For each MC run, the residuals curve is plotted, and the AON is executed with the covariance released. The steepness calculated from the residual curve is compared with the solution of AON to investigate how steepness is necessary for the AON solution to meet the required accuracy.

A. Far range rendezvous

When the chaser is far from the target, a co-elliptic approach is employed by setting the relative eccentricity vector $(a\delta e_x, a\delta e_y)$ to zero. The co-elliptic approach offers the advantage that passive-abort (PA) trajectories are safe due to the altitude difference. In this simulation, the relative trajectory is generated setting $\boldsymbol{x}(t_o)$ to $(1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 100)^T$ [km]. Regarding AON performance, the differences between CVL and RTN

Table 1. Simulation conditions

Propagator	Geopotential model: JGM3 20×20 Atmosphere model: NRLMSISE-00 Drag estimation error: ±15%
VISCAM	Noise: 0.02 deg random (1σ) Interval: 60 sec Sunblind angle: 60 deg
Residuals curve	$\mu \in [0.9 \cdot a\delta u \quad 1.1 \cdot a\delta u]$ $\boldsymbol{P}^{apr} =$ $\mathrm{diag}(1e^6, 1e^6, 1e^6, 1e^6, 1e^6, 1e^{-6})$ km
AON	$\boldsymbol{P}^{apr} = 1e^{12}\boldsymbol{I}_{6\times6}$ (No a priori info.) Requirement accuracy: ±5% of $a\delta u$

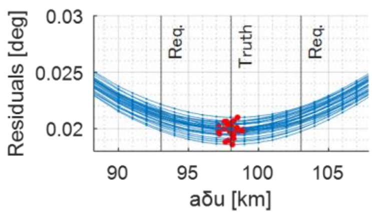


Fig. 9. Residuals curve in far range rendezvous

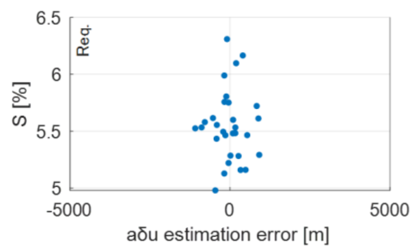


Fig. 10. The steepness in far range rendezvous

coordinates are significant, and observability is supposed to be ensured without maneuvers.

The results of the residual curve and steepness S is plotted in Fig. 9 and Fig. 10. In Fig. 9, blue dots correspond to $\boldsymbol{x}^{\mu}$, where observation residuals are calculated, while red dots represent final solutions of AON when a-priori covariances are released. In Fig. 10, the horizontal axis represents the navigation error of $a\delta u$, and the vertical axis is steepness S. This figure demonstrates that the steepness is sufficient, and the performance of AON meets the requirement accuracy; $a\delta u = 100 \pm 5\text{km}$.

B. *Middle range rendezvous*

As the relative distance becomes shorter, the CVL and RTN coordinates become identical systems, and maneuvers are needed to stimulate observability. When the relative distance is large, changes in LOS angles due to maneuvering are small. Consequently, ensuring observability is the most challenging task in the middle range scenarios.

Referring to the approach trajectories of ARGON and AVANTI, the chaser approaches along shrinking safety ellipse (SE) trajectories like Fig. 8. A SE trajectory forms a circle in the radial-normal plane. While executing maneuvers to reduce the diameter of this circle, the chaser approaches the target. Rendezvous with a SE trajectory offers the advantage that the PA trajectory can safely loop through the target.

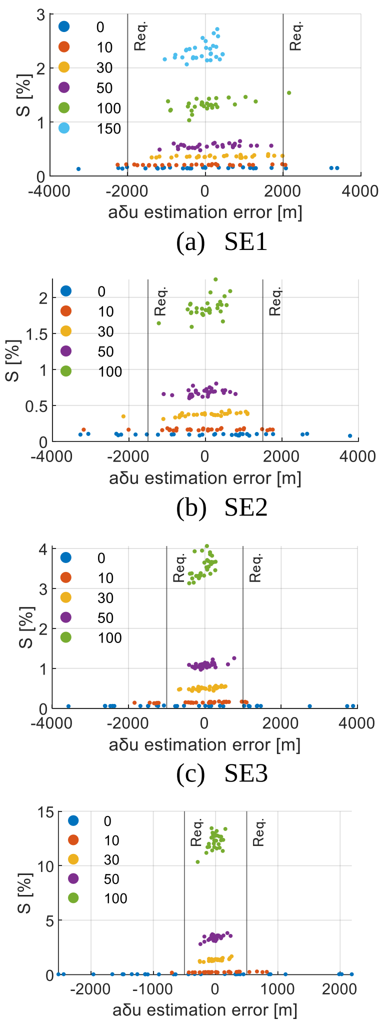
SE trajectories are simulated by setting initial states $\boldsymbol{x}(t_o)$ to $(0 \quad 0 \quad a\delta e_y \quad 0 \quad a\delta i_y \quad a\delta u)^T$, and the required amounts of maneuvers to ensure observability are investigated. Initial state conditions are summarized in Table 2. $a\delta a$ is set to zero for all cases in this scenario, and the chaser does not drift towards the target. Maneuvers consist of three: a single maneuver in the Normal direction to change $a\delta i_y$, and a pair of maneuvers in the Tangential direction to change $a\delta e_y$. The maneuver estimation error is 1% (1σ), and all other simulation conditions are the same as far-range simulations.

For each SE trajectory, 30 MC runs are conducted, setting various maneuver sizes. The steepness S calculated from the residuals curve is shown in Fig. 11. In the plots, labels represent the size of maneuvers. For example, "100" indicates maneuvers to shrink SEs by 100 m.

Fig. 11 indicates that when changes in SE are small, the observability is insufficient, and the accuracy of AON is poor. Residual curves are plotted in Fig. 12 for cases where SE1 with "0," indicating no maneuvers, and with "150," indicating shrinking SEs by 150 m. The residual curves are almost flat without maneuvers, while they are steep in the case with maneuvers. Comparing the results from SE1 to SE4 indicates that when the relative distance is larger, more maneuvers are necessary. The steepness S of 2 to 3% indicates that the observability is sufficient for the required accuracy of AON.

Table 2. SE conditions

	SE1	SE2	SE3	SE4
$a\delta e_y$, $a\delta i_y$	500 m	400 m	300 m	200 m
$a\delta u$	40 km	30 km	20 km	10 km



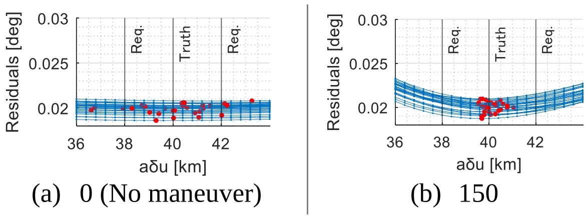
(a) SE1

(b) SE2

(c) SE3

(d) SE4

Fig. 11. The steepness in middle range rendezvous



(a) 0 (No maneuver)

(b) 150

Fig. 12. Residual curve in SE1

C. *Close range rendezvous*

When the chaser approaches the target closely enough, the navigation sensor used is switched, for example, to an IR camera, as shown in Fig. 1. If a handover operation takes time due to trial and error, the AON needs to maintain the

SE trajectory. As mentioned earlier, maneuvering stimulates observability at close range. From the view of fuel consumption, it is desirable to ensure observability using as few maneuvers as possible. The initial state $\boldsymbol{x}(t_o)$ is set to $(0 \quad 0 \quad 0.1 \quad 0 \quad 0.1 \quad 1)^T$ [km], and, as in the Middle range simulations, the size of maneuvers needed to stimulate observability is investigated. The simulation conditions are the same as those for the middle range.
However, to plot the residuals curve, $\boldsymbol{P}^{apr}$ is changed to $\mathrm{diag}(1e^4, 1e^4, 1e^4, 1e^4, 1e^4, 1e^{-10})$, considering the short relative range. Fig. 13 represents the steepness calculated from the residuals curve. Comparing the results to those in the middle range, a relatively large steepness, about 10%, is necessary to meet the required accuracy.
We doubt the atmospheric density error is the reason for these results. The atmospheric density error is set to zero for an ideal situation, and the results of Fig. 14 are obtained. In this simulation, no maneuver is executed to enhance observability. However, the steepness has sufficient values, and the AON meets the required accuracy. The atmospheric drag works as a Tangential maneuver if its estimation is perfect. Further, the changes in LOS angles caused by the drag are larger as the relative range is closer. However, estimating the atmospheric drag is difficult, which results in estimation errors. If the estimation error is significant, it worsens the performance of AON. As a result, these simulations required larger maneuvers to exceed the contribution of the estimation error of atmospheric density.

V. Conclusions

In non-cooperative rendezvous missions, Angles-Only Navigation (AON), which estimates relative trajectory using LOS angles only, is a critical navigation method. The challenges of AON are its weak observability and the integrity assessment method, which was researched in this paper.
This research represented that the steepness of the residuals curve is an efficient index for assessing the AON's observability. In far and middle range rendezvous scenarios, if the increment in residuals against the minimum of the residuals curve reached a few percent, it was found that sufficient observability was achieved. However, at close range where the estimation error of atmospheric density strongly influences, a steepness of 10% calculated from the residuals curves is needed due to its estimation error.
At close range, maneuvers are employed to stimulate obserevability, thus the sensitivity to estimation bias from observation noise, modelling errors, etc., is relatively small. However, since these error factors have limited impact on observability, i.e. the steepness of the residuals curve, it is difficult to discern from the residuals curve. For future works, we will consider assessing the errors that may slightly worsen such estimation results.

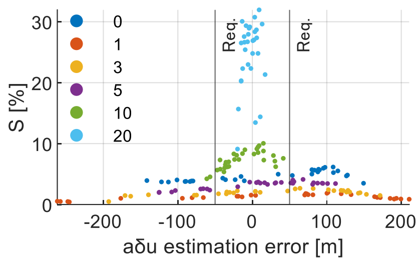


Fig. 13. The steepness in close range rendezvous

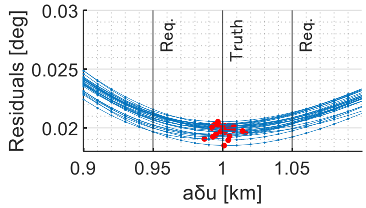


Fig. 14. Residuals curve in cases without drag estimation error

VI. References

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