Ballistic Lunar Transfer Design with Constraints on the Arrival Orbital Plane

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***Abstract* – ispace, a global lunar resource development company with the vision, "Expand our Planet. Expand our Future.", specializes in designing and building lunar landers and rovers. ispace aims to extend the sphere of human life into space and create a sustainable world by providing high-frequency, low-cost transportation services to the Moon. As part of that vision, Mission 1 (M1) was launched on December 11th 2022 carrying several commercial payloads to the Moon. One of the characteristics of this mission was a strict requirement of the landing time in order to align the surface operations phase with the local day on the surface. Typically, this can be achieved with a careful targeting of the arrival B-plane geometry. However, in order to minimize the required propellant and maximize the payload mass, a low energy transfer (LET) leading to a ballistic capture around the Moon was employed. In this scenario, the B-plane does not exist when the osculating arrival orbit is elliptical, and another method is needed. In this paper, we present a design approach that enables targeting a landing site at a particular time, arriving at the Moon through a low energy transfer following an Earth launch with optional lunar flybys. First, Keplerian dynamics are used as an initial guess of the Low-Lunar Orbit (LLO) from the Lunar Orbit Insertion maneuver (LOI) until the De-Orbit Insertion maneuver (DOI) at the right location and time. Then, a grid search on some of the key parameters of the pre-LOI arrival conditions is performed, and each sample of the solution space is propagated backwards in time. We prune these results looking for trajectories that arrive to the Moon through the vicinity of the Earth-Moon L2 point and depart from the vicinity of the Earth or have another close encounter with the Moon. We identify families of trajectories that are likely to support a wide launch window, and then optimize under a high-fidelity model using a nonlinear optimization library. Afterwards, the LOI is divided into smaller maneuvers, numerically optimized and the whole trajectory is adjusted for continuity between the transfer and the lunar arcs. Finally, the whole launch window is designed by fixing the pre-LOI conditions and varying the launch time. This approach ensures that the candidate trajectories launch from Earth and arrive at the Moon following a LET, finally landing in a particular location on the Moon at a specified time.**

I. INTRODUCTION

ispace, a global lunar resource development company with the vision, "Expand our Planet. Expand our Future.", specializes in designing and building lunar landers and rovers. ispace aims to extend the sphere of human life into space and create a sustainable world by

Fig. 1. ispace M1 lander

providing high-frequency, low-cost transportation services to the Moon. As part of that vision, Mission 1 was launched on December 11th, 2022 on board a Falcon 9 en route to the Moon. Fig. 1 shows ispace M1 lander.

A Direct Transfer (DT) to the Moon is the most straightforward way for a satellite to depart the near-Earth region and be captured into the gravity well of the Moon. It has been used multiple times since the dawn of spaceflight by the Luna, Zond, Ranger, Surveyor and Apollo projects [1]. In more recent times it has been used by the Japanese Selene/Kaguya mission, the Chinese Chang'e and Indian Chandrayan programs, as well as by private companies such as SpaceIL (Beresheet), Intuitive Machines and Astrobotics. With this strategy, the spacecraft is put into an orbit that intercepts the Moon in about 2 to 5 days, with a relatively high arrival-speed. Since the seleno-centric arrival orbit is hyperbolic, it is relatively easy to choose which orbital plane to inject into by means of an upstream b-plane targeting maneuver. In turn, this orbital plane defines the landing opportunities at a particular location, once every time the spacecraft flies close to the zenith of the landing site.

As an alternative, Ballistic Lunar Transfers (BLT), also called Low-Energy Transfers (LET) or Weak Stability boundary trajectories (WSB), exploit the gravity of the Sun to reduce the delta-v required to inject into lunar orbit by about 100 to 150 m/s [1]. In these trajectories, the spacecraft travels far beyond the lunar orbit distance up to 1 to 1.5 million km during several months, which effectively reduces the energy at lunar arrival if the geometry in the Sun-Earth system is correct. In fact, the arrival energy can become negative in a ballistic capture case, which can hinder the applicability of traditional orbital plane techniques described above. Examples of missions that have used a LET are the Japanese Hiten spacecraft, the American GRAIL mission or the Korean Danuri spacecraft.

In this work, we present a framework to generate Low-Energy Transfers to the Moon launching from Earth and selecting the orbital plane around the Moon, enabling selection of the landing time at the target location on the lunar surface. First, we analyze the lunar orbits that enable landing at specified times and locations on the Moon. Next, we determine the key parameters to reduce the search space of a backwards propagation to a tractable time, which provides a database of trajectories that start in the proximity of Earth and arrive at the Moon with the required orbital plane and low specific orbital energy. We then identify members of this database that are likely to generate a wide launch window. The process is finalized with the numerical optimization of the trajectory for all days of the launch window, which to simplify operations are designed with a shared lunar orbital phase.

II. DESIGN APPROACH

A. Lunar Orbit Analysis

One of the mission objectives of the ispace M1 spacecraft is to land close to the local sunrise at the landing site to maximize the length of the surface operations, which are constrained to one lunar day. Additionally, the landing should start in a circular 100x100 km altitude LLO. Another important limitation imposed by the lander bus is that the eclipse duration must be below about a third of the orbital period of the target orbit.

Given the latitude λ and longitude ϕ of the landing site, we can derive a relationship between the longitude of the ascending node Ω and orbital inclination i of the LLO through the flight path azimuth χ :

$$\cos i = \sin \chi \cos \lambda, \quad (1)$$

$$\Omega = \phi - \operatorname{atan2}\left(\sin \lambda \sin \chi, \cos \lambda\right), \quad (2)$$

where all quantities are expressed in a Moon-fixed frame at the time of landing. Geometrically, this constraint means that the orbital plane must contain the landing site.

Fig. 2 shows the relationship between Ω and i for $\lambda \simeq 40\,\mathrm{deg}$ and $\phi \simeq 34\,\mathrm{deg}$.

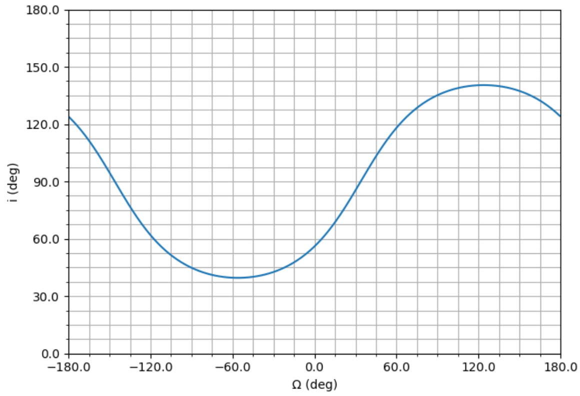

Fig. 2. Inclination as a function of the longitude of node to land on a particular lunar location

A direct consequence of this relationship is that the ground-track has to be able to cover the landing site; mathematically this can be written as

$$\frac{\pi}{2} - |\phi| \geq i \geq \frac{\pi}{2} + |\phi|. \quad (2)$$

If this condition is satisfied, and excluding degenerated cases, there are usually two possible orbital planes for a fixed value of the inclination, one in which the landing happens towards the ascending node, and another one towards the descending node. The exception to this rule are landing sites exactly on the poles, which are accessible by every polar orbit (infinite solutions), or on the equality signs of (1) which yield only one possible orbital plane.

The 100x100 km altitude state can then be backwards-propagated in time. Due to the non-spherical gravity of the Moon, this will cause the eccentricity vector to evolve with time. However, this effect can be minimized if a polar LLO is chosen. Another benefit of choosing the inclination near 90 deg is that the eclipse requirement can be satisfied for a longer period of time. To the first approximation, the longitude of node in the Moon-fixed frame will evolve linearly with time owing to the rotation of the Moon at a rate of about 13 deg per day.

When considering the Lunar Orbit Insertion (LOI) maneuver to capture an incoming trajectory into the target LLO, the required delta-v can be written as

$$\Delta v_{\mathrm{LOI}} = \sqrt{\left(v_p^*\right)^2 + \Delta c_3} - v_p^*, \quad (3)$$

where $v_p^* \simeq 1.6335\,\mathrm{km/s}$ is the orbital speed of a 100x100 km LLO, and c_3 is two times the specific energy. Delta-v efficient LETs can achieve negative values of c_3 (ballistic capture), typically on the order of $\mathcal{O}\left(0.1\,\mathrm{km}^2/\mathrm{s}^2\right)$. Equation (3) can be expanded on Taylor series around zero to provide a very good approximation:

$$\Delta v_{\mathrm{LOI}} \simeq 0.677 + 0.2164\, c_3\,\mathrm{km/s}, \quad (3)$$

with an error smaller than $1\,\mathrm{m/s}$ for c_3 values up to $\pm 0.25\,\mathrm{km}^2/\mathrm{s}^2$. Thus, lower pre-LOI c_3 values are preferred in order to minimize the magnitude of the lunar insertion cost.

B. Backwards Search

In general, performing a backwards search without any pre-pruning of all potential transfer trajectories would be too much time consuming, as the number of candidates grows exponentially with the dimension of the search space. However, we can do a series of simplifications to make this problem tractable.

In the first place, we limit the search only to polar orbits, since non-polar orbits are less favorable in general from the point of view of eclipses and less robust to the non-spherical gravitational perturbation of the Moon as explained in Section II.a. Next, we fix the landing site, which in turn, determines the landing time after choosing the local solar time at touchdown. We select landing towards the ascending or descending node to uniquely determine the orbital plane. Furthermore, we exploit Keplerian dynamics to assume that the longitude of node, eccentricity and semi-major axis, as well as all the other orbital elements, are constant during the LLO when expressed in an inertial reference frame. The state right before LOI can be constrained to periapsis to minimize the magnitude of the LOI maneuver.

With these assumptions, we can fix the periapsis altitude h_p, orbital inclination i, longitude of node Ω, and the true anomaly before LOI, $\nu = 0$. We are left with three free variables: the time between landing and arrival Δt, the argument of periapsis ω and the arrival c_3. A mesh in this three variables can be constructed. Since in a later stage of the design the LOI maneuver will be split in smaller maneuvers, and considering the minimum separation between maneuvers, we can define a minimum value for Δt, while the maximum bound can be set as about a month larger than the minimum bound, since spending multiple lunar days in lunar orbit is not a mission requirement and another landing opportunity could be potentially found in the previous local day. The argument of periapsis ω can be sampled uniformly in its domain of definition, but values around integer multiples of 90 deg are preferred to minimize the Lidov-Kozai mechanism induced by Earth's gravity during the intermediate orbits during the LOI sequence, which may strongly perturb the evolution of the orbital inclination and perilune altitude [2, 3], hindering the design process. Finally, the values of c_3 are chosen in a range that allows for a small LOI magnitude. Note that values of c_3 below approximately $-0.2\,\mathrm{km}^2/\mathrm{s}^2$ may prevent the spacecraft from crossing the zero-velocity curves in the vicinity of the Earth-Moon L2 point, where the exact threshold depends on the position of the Moon on its orbit, owing to its non-zero eccentricity.

All samples of the mesh grid can be then be propagated backwards in time for a maximum duration imposed by the mission requirements on the maximum transfer time-of-flight. Using parallel computing is highly recommended to speed up this task. Trajectories that do not escape the Moon or do so through the L1 region, as well as trajectories that collide with the Moon can be pruned during this process. All trajectories that reach the vicinity of the Earth are classified as potential candidates for a transfer. An additional pruning on the launch inclination can be made, keeping only trajectories whose inclination is close to the optimal inclination provided by the launch vehicle. The results of a sample search are

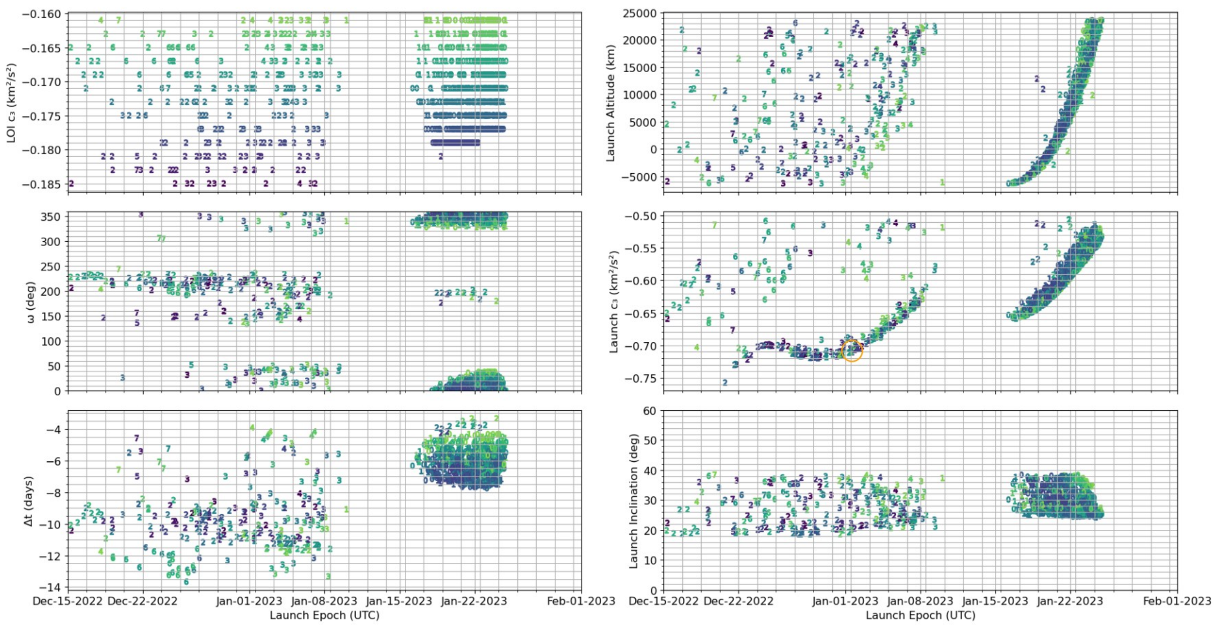

Fig. 3. Backwards propagation search results

shown in Fig. 3, which contains the pre-LOI c_3, argument of periapsis and time from landing to LOI (negative as it corresponds to a backwards propagation) in the left column, and the launch altitude, c_3 and inclination on the right column. Each trajectory is colored according to its LOI magnitude (a darker color corresponds a to smaller delta-v), and marked with the number of perilunes of the spacecraft while it is inside the sphere of influence of the Moon during its final approach.

All these trajectories can be used as an initial guess for a numerical optimization process, but it is convenient to select those trajectories that can support a wide launch window with a small delta-v added cost. For example, the cluster of trajectories on the right side of Fig. 3 is not suitable to support a wide launch window because it shows a strong dependency between the ballistic launch periapsis altitude and the launch date. When adjusting this altitude to a realistic value, a relatively large Deep Space Maneuver (DSM) is likely to appear for launch days in which the ballistic propagation launch periapsis correction is large. Additionally, a smaller number of perilunes in the final approach is also desirable to be able to easily adapt the final trajectory. With these considerations, an initial good choice would correspond to the trajectory launching around 2023-01-01 and launch c_3 of about $-0.7\,\mathrm{km}^2/\mathrm{s}^2$ (orange circle in the mid-right subplot of Fig. 3).

C. Lunar Orbit Design

The lunar arrival state from the backwards search is used to design the lunar orbit phase, which spans from lunar arrival to the De-Orbit Descend maneuver (DOI). The LOI maneuver is split into three smaller maneuvers to minimize gravity losses and increase the robustness of the trajectory to execution errors and anomalies. The period after LOI1 and LOI2 are set to approximately 8 and 4 hours, respectively, while the period after LOI3 is about 1.96 hours, which corresponds to a 100x100 km altitude lunar orbit.

LOI1 and LOI2 are planned with a relatively small separation to minimize the effects of the Lidov-Kozai mechanism on the intermediate orbit. These perturbations can induce a drift on the periapsis altitude that could require a sub-surface arrival to reach a final 100x100 km orbit, which clearly leads to a non-feasible trajectory. Due to eclipse constraints, LOI3 must be relatively close to DOI, with the exact limit depending on the landing local solar time and the exact admissible maximum eclipse. Small variations of the arrival state are allowed during the numerical optimization problem, which is run in multiple steps, increasing the fidelity of the dynamical model in each iteration.

Fig. 4 shows a typical lunar orbit with its LOI sequence.

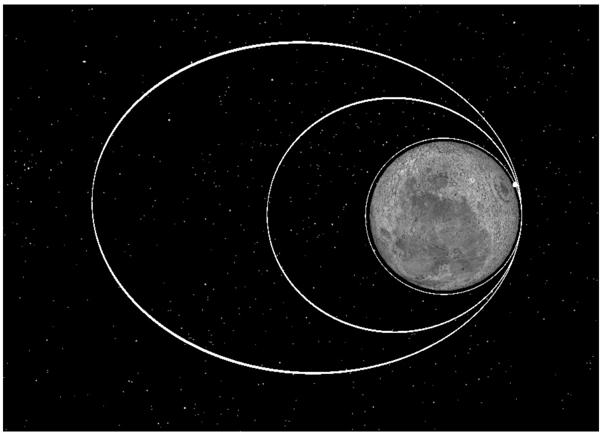

Fig. 4. Lunar Orbit Insertion sequence

D. Detailed Transfer Design

The next step is to adjust the backwards-propagated candidate trajectory to make it compatible with the launch geometry and the modified arrival state.

To this end, we employ a multiple-shooting algorithm in which we allow variations of the state vectors at launch, apogee, lunar approach and pre-LOI, and numerically match the position at intermediate epochs. In the matching points, we allow velocity discontinuities that are directly mapped to DSMs. The time of these matching points is also part of the control variables. Similar to the LOI design, this process is also performed with incremental complexity of the dynamical model.

To reduce the magnitude of the DSMs, some flexibility can be introduced in the LOI1 time, at the expense of iterating the lunar orbit design of Section II.C.

E. Launch Window Generation

Finally, we generate trajectories for all days in the launch window. To reduce the complexity of the design process, and to simplify the preparation and execution of the real-time operation of the spacecraft, the lunar orbit design is considered fixed for the whole launch window of a particular month, and only variations of the arrival energy of the pre-LOI state, with fixed perilune radius, are allowed since they can be absorbed by fine-tuning the LOI1 maneuver.

If the transfer for one day of the launch window is available, it can be used as initial guess for a contiguous day, as long as the initial geometry is still adjusted to the launcher requirements. This process yields one trajectory per launch day of the window, as shown in Fig. 5.

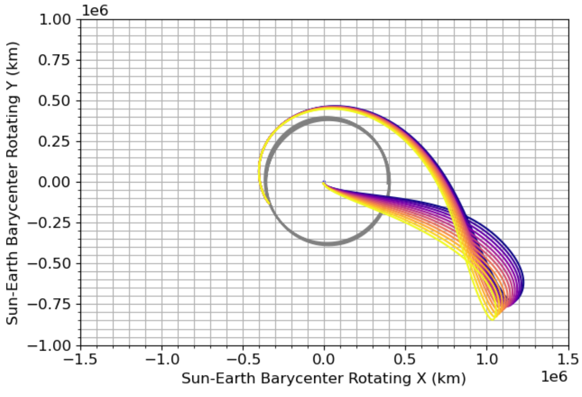

Fig. 5. Launch window trajectories in the Sun-Earth rotating frame

III. CONCLUSIONS

A method to design Low-Energy Trajectories departing from Earth and arriving at the Moon with a particular orbital plane is presented. This approach allows for a spacecraft to land at a specified time on the lunar surface, and was used in the first ispace lunar mission M1.

The approach hinges on carefully selecting key parameters of the resulting lunar orbit to construct databases of candidate trajectories that launch from Earth. Candidates that could support a wide launch window are selected and their transfer and lunar orbits are optimized numerically. Finally, the launch window is generated. Several operational considerations are embedded in the process to reduce the workload of the staff preparing the mission and to reduce the probability of errors during flight.

IV. REFERENCES

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