A Novel Approach for Tracklet-Object Association Using a Pseudo-Probability Metric

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***Abstract* – Passive-optical observations of resident space objects result in tracklets with astrometric angle pair measurements. These tracklets are checked against a base catalogue for possible associations to already known objects. The association process may be ambiguous, as the measurements in tracklets carry uncertainties and/or satellites may have manoeuvred. This ambiguity can either occur in the form of an association of one tracklet to multiple objects or of multiple tracklets to one object. Furthermore, it is possible that no catalogue object can be identified for a given object, even if a tracklet was the result of a follow-up observation. All of these possibilities may lead to misleading associations or none at all, and thus the orbital information of the corresponding objects may not be updated. Commonly, the tracklet-object association is performed by calculating the Mahalanobis distance between the measurements and the propagated positions of the objects.**

Within the sensor network SMARTnet (Small Aperture Robotic Telescope Network) and the attached database BACARDI (Backbone Catalogue of Relational Debris Information), a new approach is applied: each tracklet-object combination is assigned a number reflecting a likelihood value for this combination to be true.

This value is derived by comparing the measured positions to propagated positions based on catalogue orbits. The deviation from a perfect match is derived via coordinates in a normalized vector space and the norm, i.e., the distance to the origin, is converted into the likelihood value, called pseudo-probability. The pseudo-probability values of all possible tracklet-object associations for a given tracklet are stored. In the subsequent orbit determination process, the measurement residuals within each tracklet are assessed for all candidate tracklet-object combinations, and the combination resulting in the lowest residual RMS is selected. In case of all residuals being too large, the tracklet will be stored without any association in the database for further analysis.

We will describe the method in detail and show the advantages compared to currently used methods. In addition, we will discuss some challenges that have to be overcome for a fully automated process.

I. INTRODUCTION

The German Aerospace Center (DLR) and the Astronomical Institute of the University of Bern (AIUB) co-host SMARTnet, a network of passive-optical telescope stations (see [1] with updates in [2]). As of 2024, the consortium consists of more than 10 globally distributed telescopes. These telescopes are operated by DLR, AIUB, and several partner organisations. Tracklets resulting from observations of these telescopes converge at DLR and are distributed amongst the partners. These tracklets must not be associated before inserting them into the network due to export regulations. An object association process against a base catalogue must therefore be a subsequent process. For observation series, where only one object was observed and only one tracklet resulted, the association might be unambiguous. However, when observing satellite clusters, certain base catalogues, e.g., Two-Line Element (TLE) sets, may already contain incorrectly associated objects [3]. Extending the task to fragmentation events, ambiguities can hardly be avoided, especially in early stages after an event, because of short arc orbit determination and propagation. Inherent difficulties with short arcs are presented and described in [3].

In the following sections, we will describe an approach developed at DLR to increase the number of correctly associated tracklets per night. We will lay out the idea behind this approach and discuss the benefits compared to other methods.

II. THEORETICAL BACKGROUND

A. Distance Measure

The newly developed algorithm has its roots in the Mahalanobis distance, which is calculated via a covariance matrix S [4]. The Mahalanobis distance may be written as:

$$D(\vec{x}, S) = \sqrt{(\vec{x} - \vec{\mu})^T S (\vec{x} - \vec{\mu})}, \qquad (1)$$

where $\vec{x}$ represents measurements and $\vec{\mu}$ denotes reference values.

In case of the observation association process, $\vec{x}$ represents derived values from the passive-optical measurements and $\vec{\mu}$ denotes the corresponding values based on the base catalogue.

Based on [2], we decided in favour of three independent values to evaluate the distance measure:

(i) Angular separation of the mean position with respect to a base catalogue object
(ii) Difference in the magnitudes of the angular velocities
(iii) Angle between the angular velocity vectors

We modify the matrix S in (1) to represent a diagonal normalization matrix:

$$\hat{S} = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}, \tag{2}$$

with parameters taken from [5]. Those parameters are based on statistical analyses of successfully associated tracklets. A successful association is achieved when the associated object is confirmed by a subsequent orbit determination.

The matrix $\hat{S}$ in (2) is not a covariance matrix anymore, therefore the distance D cannot be called Mahalanobis distance. However, the value of D still represents a distance measure in the $\mathbb{R}^n$ vector space.

The calculation for a single tracklet can be performed with all objects in the base catalogue leading to "distance" values for each object.

The vectors $\vec{x}$ and $\vec{\mu}$ are not limited to the three independent value stated above. The may be extended or exchanged together with adapting the matrix $\hat{S}$.

B. Pseudo-Probability Metric

To get a more pronounced representation, we convert the distance measures into a likelihood, which we call pseudo-probability:

$$p = 1/(D + 1) \tag{3}$$

Values of p always lie in the interval $(0, 1]$. The distinction from a true probability is that for a single tracklet the pseudo-probability values summed up over the entire catalogue are not necessarily equal to one, which is due to the incompleteness of the publicly available catalogue. Other functions than (3) are possible, however, as long as they are strictly decreasing, the conclusions are comparable.

C. Decision Matrix

In general, the association of tracklets to objects is unambiguous: each tracklet belongs to a single object – may it be already catalogued or not – and an object can only be represented by a single tracklet of a given series of observations. Often, there are multiple tracklets per series if several objects lay in the field of view. This fact can be used to evaluate the candidate associations.

The pseudo-probability values for all tracklets from a given series may be displayed in a matrix as shown in Tab. 1. Columns represent the tracklets of the observation series, while rows show the catalogue objects. For the sake of simplicity, four tracklets and the pseudo-probability values of their associations to five catalogue objects are shown here. The most probable matches are highlighted with a green background.

Table 1. Example of a decision matrix.

		Tracklet			
		1	2	3	4
Object	A	0.99	0.50	0.21	0.10
	B	0.32	0.51	0.29	0.31
	C	0.68	0.49	0.97	0.45
	D	0.27	0.40	0.25	0.91
	E	0.11	0.99	0.12	0.41

In Tab. 1 with its random numbers, there are tracklets corresponding to objects A, C, D, and E. Object B is not associated to a tracklet in this example. In theory, it is also possible for the number of tracklets to exceed the number of objects in a base catalogue, as this catalogue might be incomplete.

III. Tests with Simulated Tracklets

To prove the unambiguity of the associations, clusters represent a good test case. First, we evaluated simulated tracklets based on TLEs. The time steps between two sets of measurements are 20s with seven measurements per tracklet. A measurement in this regard consists of a right ascension / declination angle pair. There was no noise added. The calculation of the pseudo-probability values is based on the same TLE data, thus representing an ideal case. One would expect that the target objects each receive a pseudo-probability of one, while the values for other objects in the catalogue are significantly lower.

We selected the geostationary satellites ASTRA 1KR (06012A), ASTRA 1L (07016A), ASTRA 1M (08057A), and ASTRA 1N (11041A), which form the ASTRA 1 satellite cluster. They are located at 19.2°E. Additionally, we added an ARIANE 5 rocket body (08030C), because it was observed in the same images as the ASTRA satellites on 2023-11-04. It serves as an object that is occasionally close-by but does not belong to the cluster. We created a tracklet for each object in the same time interval. This represents an observation series, where one object is targeted and the others are also observed due to the relatively wide field of view. Tab. 2 shows the results of the 1st test with simulated tracklets.

Table 2. Simulated tracklets, 1st test,
epoch 2023-11-04.

	Tracklet				
	1	2	3	4	5
08030C	1.00	0.05	0.00	0.00	0.00
06012A	0.05	1.00	0.77	0.55	0.51
07016A	0.05	0.77	1.00	0.60	0.56
08057A	0.05	0.55	0.60	1.00	0.86
11041A	0.05	0.51	0.56	0.86	1.00

It is visible that the target objects show the highest pseudo-probability values for their designated tracklets and that the values for the other objects drop significantly. Furthermore, the object not in the satellite cluster shows even smaller values. Angular separation and differences in angular velocities lead to a good distinction.

This test was repeated several times with different TLE sets for different observation dates. These were chosen to display a variety of different TLE bases. The results are comparable and shown in Tabs. 3 to 5.

Table 3. Simulated tracklets, 2nd test,
epoch 2023-10-29.

	Tracklet				
	1	2	3	4	5
08030C	1.00	0.00	0.00	0.00	0.00
06012A	0.00	1.00	0.74	0.64	0.65
07016A	0.00	0.74	1.00	0.62	0.64
08057A	0.00	0.64	0.62	1.00	0.92
11041A	0.00	0.65	0.64	0.92	1.00

Table 4. Simulated tracklets, 3rd test,
epoch 2023-11-08.

	Tracklet				
	1	2	3	4	5
08030C	1.00	0.00	0.00	0.00	0.00
06012A	0.00	1.00	0.86	0.75	0.78
07016A	0.00	0.86	1.00	0.68	0.76
08057A	0.00	0.75	0.68	1.00	0.80
11041A	0.00	0.78	0.76	0.80	1.00

Table 5. Simulated tracklets, 4th test,
epoch 2023-11-11.

	Tracklet				
	1	2	3	4	5
08030C	1.00	0.00	0.00	0.00	0.00
06012A	0.00	1.00	0.81	0.85	0.60
07016A	0.00	0.81	1.00	0.82	0.68
08057A	0.00	0.85	0.82	1.00	0.66
11041A	0.00	0.60	0.68	0.66	1.00

The pseudo-probability values show very consistent behaviour:

- The target object receives a value of 1.00.
- Objects in the same cluster receive values that are significantly smaller.
- Objects not belonging to the cluster have values smaller by orders of magnitude or even equal to zero.

How much the values drop for objects within the cluster depends on proximity of the individual satellites at a given epoch.

IV. TEST WITH REAL TRACKLETS

The simulated tracklets used as a proof-of-concept in the preceding section did not carry measurement noise. They were merely used to show how the pseudo-probability values for the target objects differ compared to objects in the same cluster and objects not belonging to the cluster, respectively.

Possibly occurring difficulties will arise in the application to real tracklets depending on pointing accuracy, field of view, and measurement noise, amongst other influences. The field of view in particular will lead to additionally observed objects.

We selected two observation series, both acquired with the telescope SMART-01-B-SUTH (see [6] for details) on 2023-11-04, and calculated the pseudo-probability values like before. The results are shown in Tab. 6 and 7.

Table 6. Real tracklets, 1st series

	Tracklet				
	1	2	3	4	5
08030C	0.84	0.05	0.05	0.05	0.05
06012A	0.05	0.87	0.76	0.54	0.51
07016A	0.05	0.83	0.93	0.60	0.56
08057A	0.05	0.57	0.61	0.94	0.87
11041A	0.05	0.53	0.57	0.83	0.93

Table 7. Real tracklets, 2nd series

	Tracklet				
	1	2	3	4	5
08030C	0.88	0.05	0.05	0.05	0.05
06012A	0.05	0.89	0.76	0.54	0.51
07016A	0.05	0.85	0.97	0.60	0.56
08057A	0.05	0.57	0.61	0.98	0.85
11041A	0.05	0.53	0.57	0.87	0.98

In both matrices, the highest values are again marked in green. Especially for tracklet 2 in both cases, the two highest pseudo-probability values differ by 0.04, which leads to the conclusion that both associations might be possible, depending on the outcome of each orbit determination. This is expected to happen occasionally and is covered in the next section.

V. EDGE CASES

The real strength of this method occurs when a tracklet has high pseudo-probability values for two different objects in a catalogue and the association is ambiguous. Tab. 8 shows an example. These ambiguities may occur in cases of break-up events or recently performed manoeuvres in a satellite cluster. The base catalogue may not yet be updated, and the satellite after the manoeuvre may be close to other objects in the catalogue. For break-up events, the association may be difficult or even impossible at first.

Furthermore, noisy measurements or a small number of measurements in tracklets may lead to almost indistinguishable associations.

Table 8. Example of an ambiguous decision matrix

		Tracklet			
		1	2	3	4
Object	A	0.99	0.96	0.21	0.10
	B	0.87	0.93	0.29	0.31
	C	0.68	0.49	0.97	0.45
	D	0.27	0.40	0.25	0.91
	E	0.11	0.84	0.12	0.41

In Tab. 8, both Tracklet 1 and 2 show the highest pseudo-probability values for object A. Due to the unambiguity statement of tracklets above, it is not possible for both tracklets to belong to the same object. Because the pseudo-probability values are stored for each combination, other possible associations may be checked via subsequent orbit determination when the top tier combinations fail.
In the example above, the combinations A-1 and B-2 are checked via orbit determination, and if they fail, the combinations A-2 and B-1 are checked. This represents a big difference to other methods, where lower tier combinations are not considered because those combinations are not stored.

VI. SUMMARY AND OUTLOOK

In this paper, we presented a proof-of-concept for an improved tracklet-object association process. It takes multiple possible associations – beside the top tier – into account in case the orbit determination for an association fails. We demonstrated with simulated tracklets how the method works and showed that the assumptions still hold when applying the method to real-world tracklets.

However, if we really have the case that the highest tier association fails via orbit determination, the process can get lengthy. In case of failing orbit determination processes, there is a large number of remaining candidate tracklet-object combinations and testing those will be computationally expensive. An optimization regarding computing time has not been done yet.

All objects used in this study were in the geostationary ring. An extension to other orbital regimes has yet to be tested. The comparison to established methods like the Mahalanobis distance was not part of this study and will be performed at a later stage.

VII. REFERENCES

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