ON THE USE OF THE OBERTH EFFECT

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Abstract – *The paper will present the detailed Oberth effect for several cases of planet fly-by or Sun fly-by. The index used for comparisons purpose is coming from a relation between the infinite velocity reached and the delta-V provided by the propulsion (considered as impulsive). Actually, the square of the ratio of infinite velocity by this delta-V can be used for getting quite simple equations. This allows to search for the maximum efficiency cases. Several kinds of initial orbits are considered with fly-by from circular or elliptic orbit as well as hyperbolic orbit. From circular orbit, the infinite velocity is at maximum 1.414 x ΔV (only 41% of gain, [R 1]). From elliptic orbit, the maximum can be easily exhibited and one can get much higher advantages. From hyperbolic orbit, the total infinite velocity is coming from the initial infinite velocity plus the one coming from the Oberth effect of the delta-V provided by the propulsion, so that in such case the index used for comparisons is a difference of the previous square ratio. One can get even higher advantages in such case.*
An application case has been performed for a potential mission toward the 200 AU in 25 years. Its trajectory from Earth to Jupiter and beyond will be presented and the part coming from Oberth effect will be detailed as well.

I. INTRODUCTION

The Oberth effect is a well-known effect that occurs naturally when integrating the flight dynamic equations with thrust, especially during swing-by. An old book from Hermann Oberth mentions that at that time 1929 he already foresees to fly toward the stars using that effect for being faster… Unfortunately, this effect is not enough developed in the books for being used commonly. Only few references papers are found on this topic. In the course of my own analysis, this effect was of course found but no name could characterize what happen, and nobody could make a reference to Oberth.
The paper will present the basic idea behind the Oberth effect: this is actually a reformulation of what Oberth wrote in 1929!
Further, the interest being to reach hyperbolic excess velocities (called Vinfinite) as high as possible, a basic development of the gain in Vinfinite for a given ΔV produced by the thrusters at perigee is presented.
One need to know the Oberth effect for explaining what happen during a swing-by with propulsion at perigee: this effect depends on the focus, on the initial orbit and curiously on the infinite velocity as it it developed next.

II. OBERTH EFFECT

This effect occurs when considering adding an impulsive $\overrightarrow{\Delta V}$ to the orbital velocity, for example at perigee of an orbit.

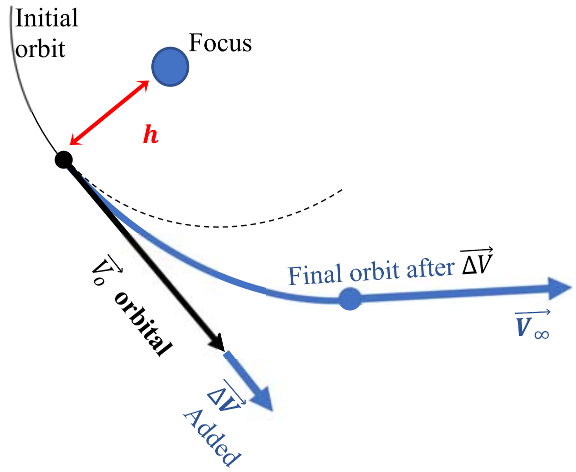

Figure 1 : Generic orbit with $\overrightarrow{\Delta V}$ added (at perigee)

The value of the final velocity depends on the altitude $\boldsymbol{h}$ of the perigee with respect to the body surface, on the value of the added $\overrightarrow{\Delta V}$ and on the orbital velocity.

A. Kinetic energy variation after a $\overrightarrow{\Delta V}$

One gets the «Oberth effect» or « deltaV perigee » or the « Oberth maneuver » always when one makes a $\overrightarrow{\Delta V}$ by using the thrusters.
One considers an initial orbit having a velocity $\overrightarrow{V_o}$ wrt its focus inertial frame. Using the classical formulation, the initial specific kinetic energy is:

$$e_{k0} = \frac{1}{2} V_o^2$$

After an impulsive $\overrightarrow{\Delta V}$ the current location on the orbit does not change, but the orbital velocity increases and the specific kinetic energy becomes:

$$e_{k1} = \frac{1}{2}(\overrightarrow{V_o} + \overrightarrow{\Delta V})^2 = \frac{1}{2} V_o^2 + \frac{1}{2}\Delta V^2 + \overrightarrow{V_o}.\overrightarrow{\Delta V}$$

So, the variation is: $\Delta e_k = \frac{1}{2}\Delta V^2 + \overrightarrow{\boldsymbol{V_o}}.\overrightarrow{\boldsymbol{\Delta V}}$

All the subtle effect comes from the scalar product term $\overrightarrow{\boldsymbol{V_o}}.\overrightarrow{\boldsymbol{\Delta V}}$ saying that the energy variation depends

on the orbital velocity $\overrightarrow{V_o}$.
When $\overrightarrow{\Delta V}$ is colinear to the orbital velocity $\overrightarrow{V_o}$, the scalar product term is maximum.
It is clear that higher the orbital velocity is, higher the energy gain is. Hence the best location to apply the $\overrightarrow{\Delta V}$ is at perigee where the velocity is the highest for the considered orbit.
Moreover, the orbital velocity increases when the perigee altitude "h" decreases: so low altitudes "h" produces the highest variation in specific kinetic energy.

III. RELATION INFINITE VELOCITY V_∞ VERSUS ΔV

The specific energy of an orbit (total energy with kinetic energy and gravitational potential) is given by the semi-major axis a_o according to the vis viva equation

> *note: vis viva equation (eq. 1) is valid for any orbit,*
> *for elliptic orbit the total specific energy is negative "$-\frac{\mu}{2a_o}$", the semi-major axis is positive,*
> *for hyperbolic orbit the total specific energy is positive, making the semi-major axis negative to keep all equations valid even in the case of hyperbolic orbits.*

Before the ΔV $\quad \frac{1}{2}{V_o}^2 - \frac{\mu}{r} = -\frac{\mu}{2a_o}$ *Eq. 1*

After the ΔV $\quad \frac{1}{2}(\overrightarrow{V_o} + \overrightarrow{\Delta V})^2 - \frac{\mu}{r} = -\frac{\mu}{2a_1}$ *Eq. 2*

with a_1 the new semi-major axis

A subtle important fact: *The location of the applied $\overrightarrow{\Delta V}$ remains identical in both equation because the $\overrightarrow{\Delta V}$ considered are impulsive, so that the radius $\boldsymbol{r}$ from the focus does not change.*
Hence, for a large enough[1] $\overrightarrow{\Delta V}$, the orbit after the ΔV becomes hyperbolic.
In eq. 2, the infinite velocity is reached when r goes to infinite and a_1 the new semi-major axis becomes negative:

$$\frac{1}{2}{V_{\infty,1}}^2 = -\frac{\mu}{2a_1}$$

So again with eq.2 $\quad \frac{1}{2}{V_{\infty,1}}^2 = \frac{1}{2}(\overrightarrow{V_o} + \overrightarrow{\Delta V})^2 - \frac{\mu}{r}$

and using eq.1 $\quad \frac{1}{2}{V_o}^2 = \frac{\mu}{r} - \frac{\mu}{2a_o}$

$${V_{\infty,1}}^2 = -\frac{\mu}{a_o} + \Delta V^2 + 2\overrightarrow{V_o}.\overrightarrow{\Delta V}$$

And for $\overrightarrow{\Delta V}$ colinear to $\overrightarrow{V_o}$ one gets the following simple equation:

$$\boxed{\frac{{V_{\infty,1}}^2}{\Delta V^2} = 1 + \frac{2V_o}{\Delta V} - \frac{1}{\Delta V^2}\frac{\mu}{a_o}} \qquad \textit{Eq. 3}$$

[1] $\overrightarrow{\Delta V}$ large enough: such that final energy is positive, i.e. $\frac{1}{2}(\overrightarrow{V_o} + \overrightarrow{\Delta V})^2 > \frac{\mu}{r}$

Note: for $\overrightarrow{\Delta V}$ anti-colinear to $\overrightarrow{V_o}$ one gets the same as eq.3: $\frac{{V_{\infty,1}}^2}{\Delta V^2} = 1 - \frac{2V_o}{\Delta V} - \frac{1}{\Delta V^2}\frac{\mu}{a_o}$ with ΔV considered as algebraic.

A. *Application for two different orbits*

To go further with what happen on the infinite velocity, it is easy to compare the result of the same $\overrightarrow{\Delta V}$ applied to two different orbits, but having the same specific orbital energy, i.e. same semi major axis a_o.
The eq. 3 shows that the ratio $\frac{{V_{\infty,1}}^2}{\Delta V^2}$ depends only on the velocity $V_o = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a_o}}$ where the same ΔV was applied, and the ratio increases with that velocity V_o (i.e. when the radius $\boldsymbol{r}$ decreases)
The extremum of $\frac{{V_{\infty,1}}^2}{\Delta V^2}$ is given for $\frac{\partial \frac{{V_\infty}^2}{\Delta V^2}}{\partial \Delta V} = 0$. After some calculus, one gets:

$$\Delta V^* = \frac{\mu}{a_o V_o} \qquad \textit{Eq. 4}$$

- For $a_o > 0$, the initial orbit is elliptic, the maximum of $\frac{{V_{\infty,1}}^2}{\Delta V^2}$ is given by:

$$\frac{{V_{\infty,1}}^2}{\Delta V^2} = 1 + 2V_o\frac{a_o V_o}{\mu} - \frac{(a_o V_o)^2}{\mu^2}\frac{\mu}{a_o}$$

$$\max\frac{{V_{\infty,1}}^2}{\Delta V^2} = 1 + \frac{a_o {V_o}^2}{\mu}$$

$$\max\frac{{V_{\infty,1}}^2}{\Delta V^2} = \frac{2a_o}{r}$$

And considering that the $\overrightarrow{\Delta V}$ is applied at perigee $r_{per} = a_o(1-e)$:

$$\max\frac{{V_{\infty,1}}^2}{\Delta V^2} = \frac{2}{1-e} \qquad \textit{Eq. 5}$$

Hence, the maximum of $V_{\infty,1}$ occurs for the most eccentric initial orbit. The higher benefit is occurring when the initial orbit is almost parabolic with a burn at perigee (r mini), *but still above the focus surface.*

Note: for a circular orbit ($e = 0$), the maximum of $\frac{{V_{\infty,1}}^2}{\Delta V^2} = 2$ i.e. max $V_{\infty,1} = \sqrt{2}\Delta V$. That is that the net benefit is 40% (exactly $\sqrt{2} - 1$).
This maximum occurs for $\Delta V = \frac{\mu}{a_o V_o}$ with $V_o =$ the circular velocity in this case. So, $\Delta V = V_{circ}$ for providing $V_{\infty,1} = \sqrt{2}V_{circ}$.
This ΔV is about twice the ΔV_{esc} needed for just escaping from a circular orbit $\Delta V_{esc} = (\sqrt{2} - 1)\cdot V_{circ}$ (but with null infinite velocity $V_{\infty,1} = 0$ in this last case).

- For $a_o < 0$, the initial orbit is already hyperbolic with its initial infinite velocity $V_{\infty,o}$, the extremum occurs for negative ΔV, so do not provide interesting extremum value.

  However, because in this case $\frac{-\mu}{a_o} = V_{\infty,o}{}^2$ the eq. 3 can be written as following:

$$\frac{V_{\infty,1}{}^2}{\Delta V^2} = 1 + \frac{2V_o}{\Delta V} + \frac{V_{\infty,o}{}^2}{\Delta V^2}$$

  In this case, the final infinite velocity $V_{\infty,1}$ includes some effects of the initial infinite velocity $V_{\infty,o}$., so, to take into account this fact, one can get the difference of their quadratic values thus:

$$\frac{V_{\infty,1}{}^2 - V_{\infty,o}{}^2}{\Delta V^2} = 1 + \frac{2V_o}{\Delta V}$$

  with $$V_o = \sqrt{\frac{2\mu}{r} + V_{\infty,o}{}^2}$$

  Note: for $\Delta V \approx 0$ one gets the maximum but limited to almost null benefit:

$$V_{\infty,1}{}^2 - V_{\infty,o}{}^2 = \Delta V^2 + 2V_o\Delta V \approx 0$$

  so eventually, for this case:

$$V_{\infty,1} \approx V_{\infty,o}$$

B. *Net benefit of using the Oberth effect*

The net benefit of using the Oberth effect is here the proportion of infinite velocity gained with respect to the amount of ΔV spent to produce the infinite velocity increase.
The general definition of the net benefit (when starting for an initial hyperbolic orbit having already an excess velocity $V_{\infty,o}$) is $\frac{(V_{\infty,1} - V_{\infty,o}) - \Delta V}{\Delta V}$.
This is a good index because when it is positive, more V_∞ variation is produced than the ΔV needed to produce it at perigee. This fully characterize the Oberth effect too. Further one can explicit the increase of infinite velocities wrt the ΔV but such equation is not easy to manipulate:

$$\frac{(V_{\infty,1} - V_{\infty,o}) - \Delta V}{\Delta V} = \frac{\Delta V}{V_{\infty,1} + V_{\infty,o}} + \frac{2V_o}{V_{\infty,1} + V_{\infty,o}} - 1 \qquad \textit{Eq. 6}$$

For the case when starting from an elliptical orbit there are no term $V_{\infty,o}$, thus the net benefit reduces to the quantity $\frac{V_{\infty,1} - \Delta V}{\Delta V}$. The net benefit of the infinite velocitie reached from elliptical orbits wrt the ΔV cost is, using eq. 3:

$$\frac{V_{\infty,1} - \Delta V}{\Delta V} = \sqrt{1 + \frac{2V_o}{\Delta V} - \frac{1}{\Delta V^2}\frac{\mu}{a_o}} - 1 \qquad \textit{Eq. 7}$$

IV. Oberth Effect starting from circular orbits

One considers here especially the cases of initial circular orbits which leads to hyperbolic trajectories after adding large enough $\overrightarrow{\Delta V}$, so that one can consider the infinite velocity $\overrightarrow{\boldsymbol{V}_{\infty,1}}$.
The comparison is based on the net benefit defined above. Not only the net benefit depends on the value of the altitude $\boldsymbol{h}$, but also on of the infinite velocity $\boldsymbol{V}_{\infty,1}$ value. It depends also on the gravitation from the focus, so the following paragraph will show the effects for 3 cases: Sun, Jupiter, Earth.

A. *Oberth Effect for the Sun*

For the Sun, it is not needed to have an altitude $\boldsymbol{h}$ very low for having a large range of net benefits due to the Oberth effect:

- Of course, for very large altitude $\boldsymbol{h}$, the Oberth effect is very low for most of the $\boldsymbol{V}_{\infty,1}$ except the very small ones (<few km/s)
- For altitude $\boldsymbol{h}$ =5.2 AU (780 million km i;e. the Jupiter altitude) there is a large range of $\boldsymbol{V}_{\infty,1}$ 10 km/s to >50 km/s which are better obtained by adding a lower ΔV.
  - For reaching $\mathbf{V}_{\infty,1}$ =18 km/s, the net benefit is 40% (up to $\sqrt{2} - 1$), i.e. the ΔV needed to be produced is only 12.8 km/s.

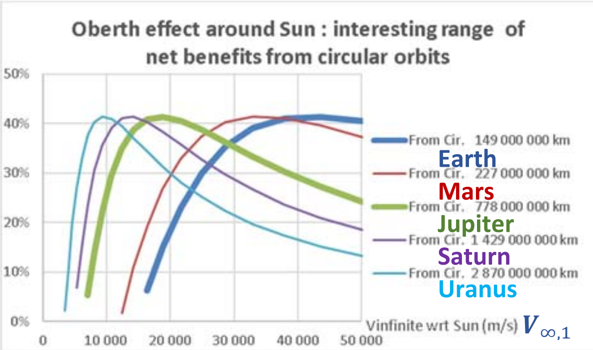

Figure 2 : Oberth effect around Sun, benefits versus $\boldsymbol{V}_{\infty,1}$ for several circular altitudes $\boldsymbol{h}$.

B. *Oberth Effect for Jupiter*

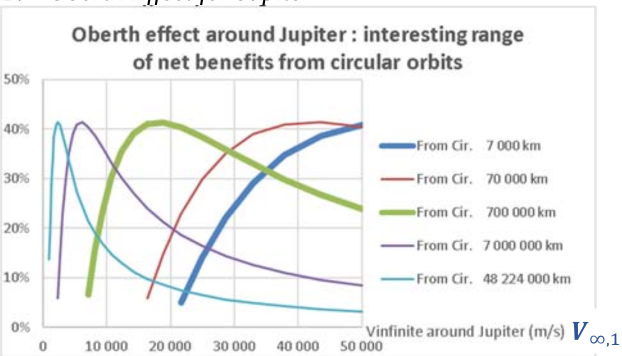

Figure 3 : Oberth effect around Jupiter, benefits versus $\boldsymbol{V}_{\infty,1}$ for several circular altitudes $\boldsymbol{h}$.

The range of altitudes considered is 7 000 km up to its sphere of influence, radius (48 million km).

- Of course, for very large altitude $\boldsymbol{h}$, the Oberth effect is very low for most of the $\boldsymbol{V}_{\infty,1}$ wrt Jupiter except the small ones (<10 km/s)
- For altitude $\boldsymbol{h}$ =700 000 km above Jupiter surface, there is a large range of $\boldsymbol{V}_{\infty,1}$ wrt Jupiter 10 km/s to >50 km/s which are better obtained by adding a lower ΔV.
    - For $\boldsymbol{V}_{\infty,1}$ =18 km/s, the benefit is 40%, i.e. the ΔV needed is only 12.8 km/s

C. Oberth Effect for Earth

- Benefits are already significant for altitude $\boldsymbol{h}$ of 300 km (above Earth Surface).
- For altitude $\boldsymbol{h}$ =300 km above Earth surface, there is a large range of $\boldsymbol{V}_{\infty,1}$ wrt Earth 7 km/s to >20 km/s which are better obtained by adding a lower ΔV.
    - For $\boldsymbol{V}_{\infty,1}$ =10 km/s, the benefit is 40%, i.e. the ΔV needed to be produced from the circular orbit is only 7 km/s
    - If the configuration of the hyperbola around Earth is well configured, i.e. the exit infinite velocity is collinear to the Earth velocity wrt the Sun (30 km/s), then the heliocentric velocity is the sum of both, that is the order of 40 km/s. But this is still a bit lower than the escape heliocentric speed of 42.4 km/s $= \sqrt{2} \cdot V_{Earth}$.

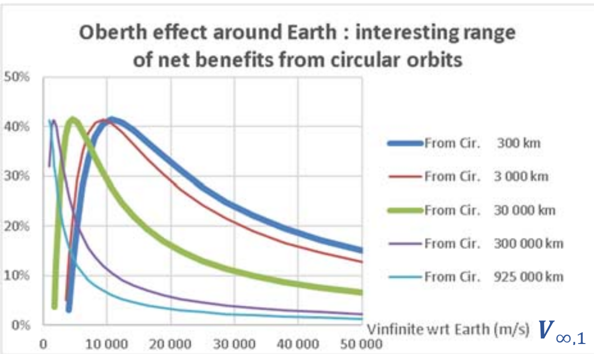

Figure 4 : Oberth effect around Earth, benefits versus $\boldsymbol{V}_{\infty,1}$ for several circular altitudes $\boldsymbol{h}$.

V. Oberth Effect starting from Elliptic or Hyperbolic orbits

This chapter deals with the same effect as for the circular starting orbit, but the benefits may be quite larger.

A. Oberth Effect for the Sun

A limited analysis has been assessed with a perigee at Jupiter orbit, and several apogees.

Huge net benefits in terms of $\mathbf{V}_{\infty,1}$ with respect to the added ΔV at perigee can come from such elliptical orbits: with only 0.3 km/s added, $\mathbf{V}_{\infty,1}$ can reach 3 km/s (a factor 10). Unfortunately, such case is not relevant for a very demanding Mission like the one toward the 200 AU in limited time (presented in the next chapter), neither operational.

However, for high energy orbits, the benefit can reach 100% which is still highly interesting.

For reaching $\mathbf{V}_{\infty,1}$ =18 km/s, the net benefit grows up to 94%, i.e. the ΔV needed to be produced (at perigee) is only 9.3 km/s for an elliptic orbit 779E+6 km x 3.11E+9 km (perigee at Jupiter distance from Sun surface, apogee at 4 time that distance).

In Figure 5, the case starting from a circular orbit (at Jupiter distance from Sun) already presented in Figure 2 is shown for comparison: that curve is now in the bottom of the plot and provides the lowest net benefits.

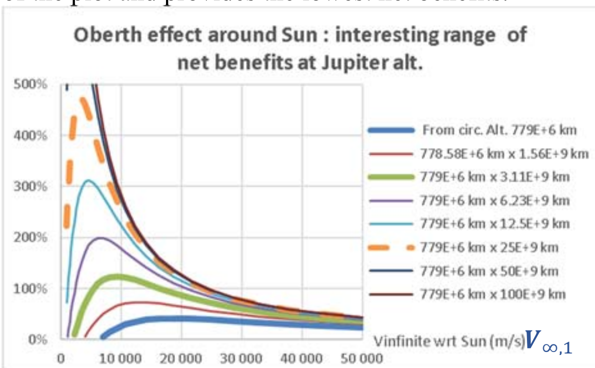

Figure 5 : Oberth effect around Sun, net benefits $\frac{V_{\infty,1}-\Delta V}{\Delta V}$ versus $\boldsymbol{V}_{\infty,1}$ for several elliptical orbits.

B. Oberth Effect for Jupiter

A limited analysis has been assessed with low Jupiter perigee altitude, and several apogees including hyperbolic (case with an entry at $\boldsymbol{V}_{\infty,o}$ =2 000 to 24 800 m/s in Figure 6).

Really huge benefits in terms of $\boldsymbol{V}_{\infty,1}$ with respect to the added ΔV at Jovian perigee can come from such elliptical and hyperbolic entry orbits.

Note: generally, for hyperbolic orbits, the maximum is reached for the given entry hyperbolic excess velocity $\boldsymbol{V}_{\infty,o}$: this is already noted above. The maximum is very high, but only valid for some small added $\|\overrightarrow{\Delta V}\| \approx 0$.

So, one can focus on the zoom Figure 6 which is still very interesting (even if not at the maximum), plotted for some fixed entry infinite velocity $\boldsymbol{V}_{\infty,o}$.

- $\boldsymbol{V}_{\infty,o}$=2 km/s, hyperbolic orbit 7 000 km x 2 km/s, with ΔV =2.8 km/s added at perigee, the exit infinite velocity $\boldsymbol{V}_{\infty,1}$ can reach 18 km/s, (see the blue dot on Figure 6). The net benefit reaches 477 %.
- The same order of benefits occurs for a stating orbit $\boldsymbol{V}_{\infty,o}$=4 km/s.

For higher entry infinite velocity $\boldsymbol{V}_{\infty,o}$ the net benefits are still > 100%.

- $V_{\infty,o}$=24.8 km/s, hyperbolic orbit 7 000 km x 24.8 km/s, with ΔV =2.8 km/s added at perigee, the exit $V_{\infty,1}$ can reach 31 km/s (see the black dot on Figure 6). The net benefit reaches 127 %.

  If the configuration of the hyperbola around Jupiter is well configured, i.e. the exit infinite velocity is collinear to the Jupiter velocity wrt the Sun (13 km/s), then the heliocentric velocity is the sum of both, that is the order of 44 km/s. This is a bit higher than the escape heliocentric speed of 42.4 km/s

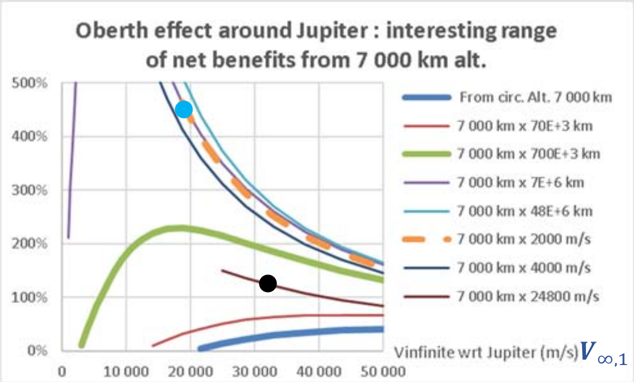

Figure 6 : Zoom of Oberth effect around Jupiter, net benefits versus $V_{\infty,1}$ ($\frac{V_{\infty,1}-\Delta V}{\Delta V}$ elliptical or $\frac{(V_{\infty,1}-V_{\infty,o})-\Delta V}{\Delta V}$ for hyperbolic entries at 2 to 8 km/s)

C. Oberth Effect for Earth

From Earth LEO or low perigee altitude, the net benefits are still impressive but for relatively low values of $V_{\infty,1}$., see Figure 7.

- Starting from a sub-GTO orbit 250 km x 22 500 km around Earth, an impulsive ΔV of 5 160 m/s at perigee will produce a $V_{\infty,1}$ of 10 307 m/s: this is a net benefit $\frac{V_{\infty,1}-\Delta V}{\Delta V}$ of 100% (dot in Figure 7).

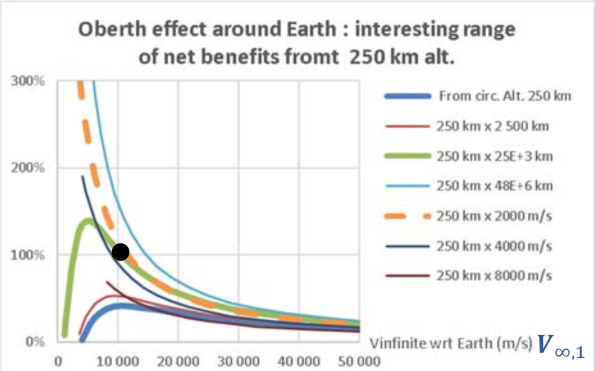

Figure 7 : Oberth effect around Earth, net benefits versus $V_{\infty,1}$ ($\frac{V_{\infty,1}-\Delta V}{\Delta V}$ elliptic or $\frac{(V_{\infty,1}-V_{\infty,o})-\Delta V}{\Delta V}$ hyperbolic)

VI. APPLICATION TO A 200 AU MISSION IN 25 Y.

The time allocation show that the velocity required is 200/25=8 AU/year that is a huge infinite velocity of 38.7 km/s. The mission considers a multi-stage vehicle launched by a standard launcher (Ariane 6). The launch is set to a sub-GTO 250 km x 22500 km.

A. First stage Hxx

A first cryogenic stage "Hxx" is used to provide a total ΔV = 5 160 m/s (in two parts). The final orbit is a hyperbolic branch with $V_{\infty,1}$ = 10300 m/s wrt Earth (first Oberth effect, black dot on Figure 6) but it is still an elliptic orbit wrt the Sun with a high apogee 1.44E9 km. Porkchop have not been considered in such first approach.

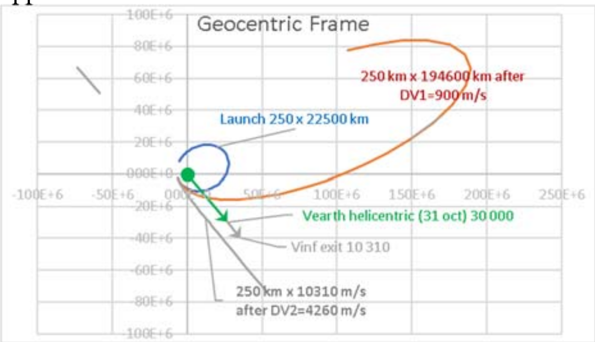

Figure 8 : First part of the 200 AU mission: "Hxx" stage function [R 5]

B. Second stage Electric Propulsion

The second stage is an Electric Propulsion System (EPS) dedicated to provide ΔV = 6 980 m/s with variable thrust consistent with the solar array power law (exponent -1.8 wrt the Sun's distance). The tool [R 4] has been used to integrate the trajectory with variable tangential thrust. The final orbit is now hyperbolic wrt the Sun. Once the continuous thrust is performed (0.7 months), the EPS stage is ejected. This final trajectory is shown on Figure 9: points "R1". and "R2". are at the intersection between the heliocentric hyperbolic orbit and Jupiter's orbit. The relevant point is however only point "R1". The velocity vector of Jupiter is plotted and is part of the velocity's vectors triangle, V IN being the arrival heliocentric velocity, "V IN=V planet + Vinf entry" define the Jovian entry infinite velocity "V inf entry".

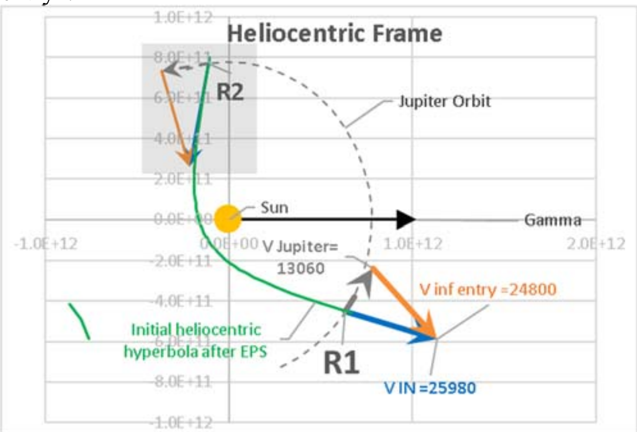

Figure 9 : Jupiter entry in the 200 AU mission [R 5]

One notices that the configuration of "V inf entry" wrt "V Jupiter" is particularly favourable for enabling a quasi-colinear infinite exit velocity with the heliocentric Jupiter velocity: this maximize the fly-by manoeuvre for increasing the velocity.

C. Third stage Jupiter swing-by

Note: The planet encounter (without thrust) can be visualized with respect to the planet center with the vector "Vinf entry" followed after the hyperbolic deviation by the vector "Vinf exit". Both velocities norms are equal (when no ΔV is added at perigee).

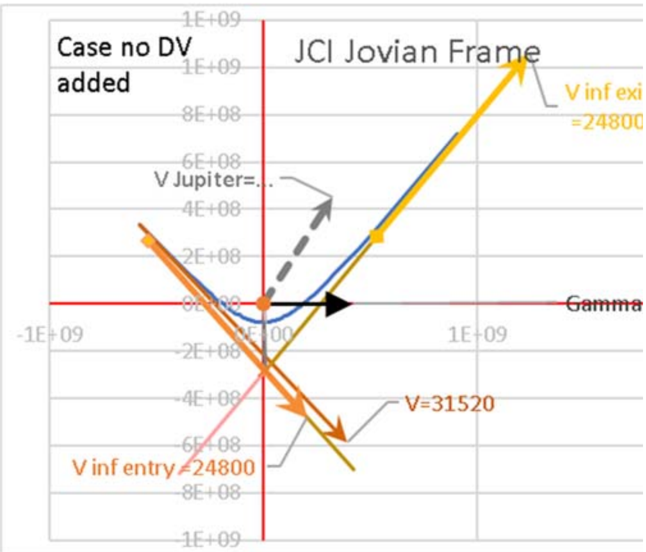

Figure 10 : Jupiter planet encounter (without ΔV) [R 5]

The third phase of the propulsion is a typical Oberth manoeuvre around the Jovian's perigee.

Once again, see Figure 12, the tool [R 4] has been used to integrate the trajectory with tangential thrust around the Jovian perigee. The final orbit is a higher hyperbolic orbit wrt the Sun.

As a summary of the planet encounter, the whole trajectory with the triangle of the velocity's vectors at entry, and in the same time the triangle of the velocity's vectors at exit is be plotted, see Figure 11. Both cases with propulsion (Oberth effect) and without propulsion are clearly shown.

The ΔV added is also shown accurately: its direction is consistent with the hyperbola perigee location shown on Figure 10 –about horizontal--.

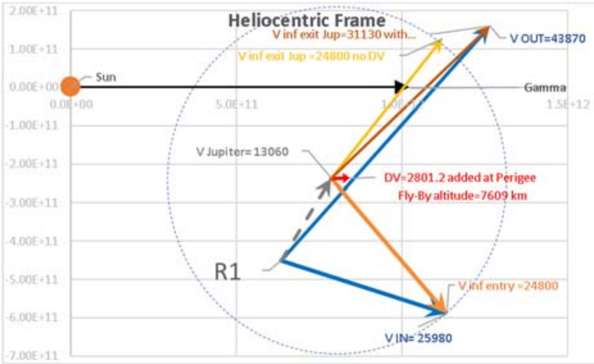

Figure 11 : Synthetic Jupiter encounter (without and with ΔV) [R 5]

However, this synthetic Jupiter encounter, Figure 11, is showing only an intermediate state. On the top of that, the final heliocentric infinite velocity "V inf helio" can be shown, keeping such synthetic plot of vectors.

With a ΔV added at perigee, the V OUT changes as well as "V inf helio".

The full synthetic plot Figure 13 "Jupiter encounter and beyond", shows the whole trajectory after the EPS function up to the final heliocentric infinite velocity to reach the 200 AU.

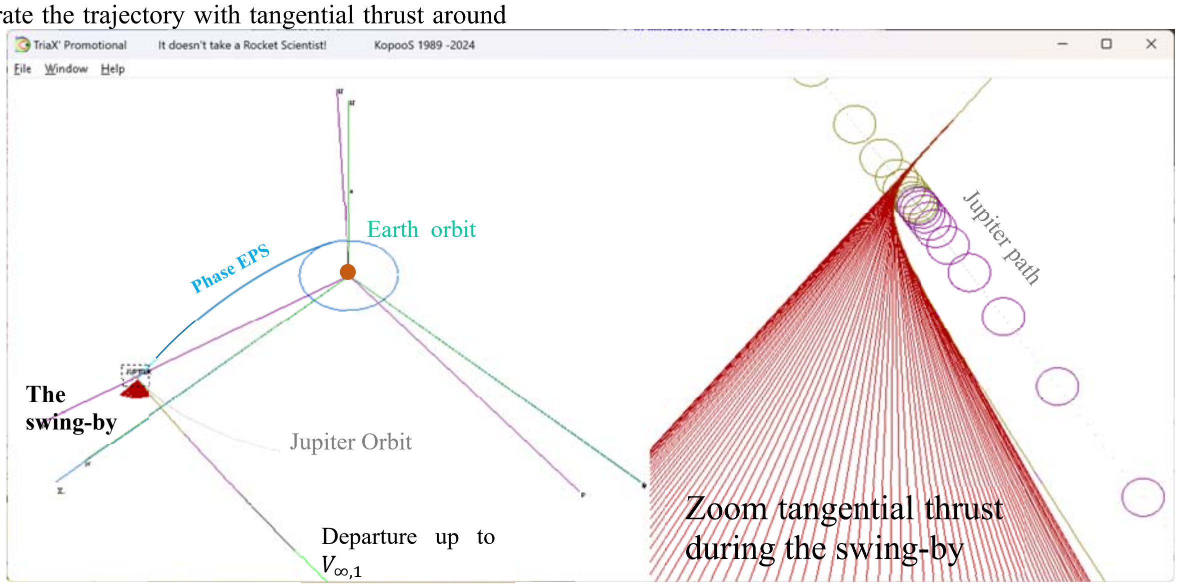

Figure 12 : Jupiter planet encounter (with thrust vector in red, zoom tangent to the trajectory)

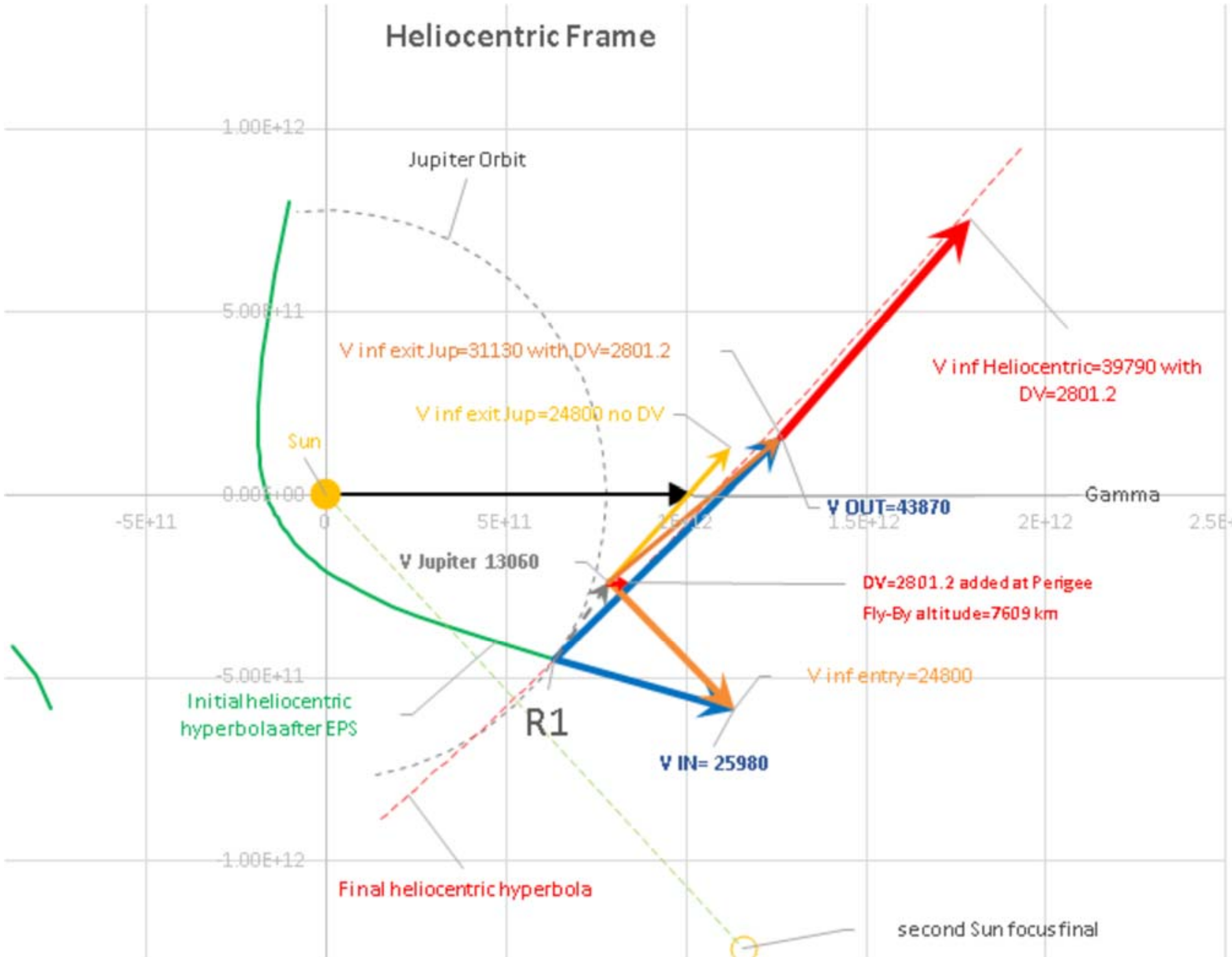

Figure 13 : Synthetic Jupiter encounter and beyond: eventually the infinite heliocentric velocity is 39.7 km/s with ΔV 2800 m/s [R 5]

Note: Figure 8 to Figure 13 are released by 3D tools [R 4] ,[R 5] *keeping the orientations along with the vector's directions and size.*

After the planet encounter, the probe follows the final heliocentric hyperbola plotted in Figure 13 starting with the velocity V OUT =43.87 m/s and reaching beyond, the final infinite heliocentric velocity of 39.7 km/s. This allows to perform the mission toward the 200 AU in 25 years.

VII. Conclusions

The Oberth effect has been detailed for several cases of starting orbit (circular, elliptical and hyperbolic), for several case of focus (Sun and planets).

Really huge Oberth effects occurs: all is explicitly coming from eq. 3 developed in § III:

$$\frac{V_{\infty,1}{}^2}{\Delta V^2} = 1 + \frac{2V_o}{\Delta V} - \frac{1}{\Delta V^2}\frac{\mu}{a_o}$$

The net benefit from the Oberth effect has been focused for reaching high infinite velocities, the index used has been defined as $\frac{(V_{\infty,1}-V_{\infty,o})-\Delta V}{\Delta V}$ and all the plots performed shows the way to enable such high benefits.

Actually, one need to know the Oberth effect for explaining what happen during a swing-by with propulsion at perigee: the paper has shown that this effect depends on the focus, on the initial orbit and curiously on the infinite velocity.

As a confirmation, two occurrences of the Oberth effect have been demonstrated in the frame of a very demanding mission toward the 200 AU in limited time (25 years).

Such mission is not easy to be undertaken, but this is also a major reason to do it, see [R 6].

VIII. References

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International Astronautical Congress, Guadalajara, Mexico, 26-30 September 2016.

[R 2] Hermann Oberth, "Wege zur Raumschiffahrt," 1925, 1928, book released 1929, (English translation NASA TT F-622, “WAYS TO SPACEFLIGHT”, 1970, https://archive.org/download/nasa_techdoc_19720008133/19720008133.pdf) (including a typo in the translation):

Relative to the system where v = 0, mass m gets an energy increase $\frac{1}{2}m\,dv^2$; relative to the moving system, on the other hand, it gets an energy increase

m.v.dv

$$\frac{1}{2}.m.(v + dv)^2 - \frac{1}{2}.m.v^2 = m.dv + \frac{1}{2}.m.dv^2 \, . \qquad (105)$$

That is considerably more. Nevertheless, the total energy in interplanetary space is not greater, for, with reference to this system, work to the amount of c.dm is simply withdrawn from mass dm.

[R 3] https://en.wikipedia.org/wiki/Oberth_effect, 2023.

[R 4] www.KopooS.com, TriaXOrbitaL tool 1989-2024

[R 5] “Module Hxx orbit06.xlsx”, internal KopooS

[R 6] Dominique Valentian et al., “Propulsion architecture enabling an interstellar medium exploration mission to 200 astronomical units in 25 years,” to be presented SP2024-171, GLASGOW, SCOTLAND | 20 – 23 MAY 2024