

# A parametric study of orbital lifetime based on semi-analytical integration—the case of INTEGRAL

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**Abstract – Highly elliptical orbits experience varying perturbations, with lunar dominance at apogee, whereas Earth's gravity and atmospheric drag at perigee. This sensitivity to initial conditions affects long-term orbital propagation, crucial for end-of-life disposal strategies like ESA's INTEGRAL mission. Diverse initial conditions result in vastly different re-entry times. Both numerical and semi-analytical techniques validate this behavior. HEOSAT employs comprehensive model perturbations, including gravitational effects and solar radiation pressure. Efficient semi-analytic propagation enables a detailed study of INTEGRAL lifetimes, revealing abrupt changes. Preliminary findings suggest that lunar effects influence INTEGRAL's orbital parameters.**

## I. INTRODUCTION

The strength of the perturbations undergone by highly elliptical orbits (HEO) switches from apogee, where the gravitational pull exerted by the Moon dominates over other disturbing forces, to perigee, where the Earth's non-central gravitation and, possibly, the atmospheric drag are the more important perturbations. A consequence of this fact may be an unusual sensitivity to the initial conditions in the long-term propagations typically needed in the design of end-of-life disposal strategies for compliance with space law [6]. This is, in particular, the case of the long-lasting ESA's Integral mission [1-4]. The wealth of highly accurate ephemeris available allows us to check that launching long-term propagations from different initial conditions of the real orbit may result in quite disparate re-entry times, ranging from just a few years to as much as a century. While this behavior nominal has been detected with the customary, highly accurate numerical integrations carried out using ESA's software tools, we checked that it can be reproduced with the high-speed and efficient semi-analytical technique. More precisely, we used our in-house mean elements propagator HEOSAT for our tests [5].

HEOSAT includes the main disturbing effects that affect the long-term propagation of HEOs, and it benefits from a robust implementation in FORTRAN77. In particular, the perturbation model considers the gravitational

effects produced by the first nine zonal harmonics of the Geopotential, including second-order effects of J<sub>2</sub>, as well as the main tesseral harmonics affecting the 2:1 and 1:1 resonances. Lunisolar perturbations are modeled in the mass-point approximation by the usual Legendre polynomials expansion of the disturbing function, which is truncated to the second degree in the case of the sun. In contrast, the accurate modeling of the lunar attraction needs to take up to the sixth-degree polynomial into account. The perturbation model also includes the effects of solar radiation pressure in the cannonball approximation and the disturbing effects of the atmospheric drag.

The efficiency of the semi-analytic propagation allows us to undertake a parametric study of the Integral lifetimes using a dense grid of initial conditions comprising up to half a million. Lifetime changes happen sharply, contrary to gradually, which makes us suspect the occurrence of successive crossings of a separatrix related to the expected Lidov-Kozai resonance due to third-body lunar effects. Moreover, preliminary results in a simplified model that only includes the Earth's oblateness and lunar perturbation indicate that the variation rates of the argument of the perigee and RAAN of Integral could be driven by the Moon's periapsis rate.

## II. NUMERICAL RESULTS

A simulation of the long-term evolution of the INTEGRAL orbit was carried out to analyze its sensitivity. The starting points are considered from the nominal mission plan prepared by the flight control team, ranging from November 2021 to November 2022. There is a one-minute spacing between the ESA J2000 position and velocity vectors, so there are 525,600 states. The propagation is done by the mean elements propagator HEOSAT. The perturbations considered include zonal harmonics, Lunisolar effects, solar radiation pressure, and atmospheric drag. The cross-sectional area value is 29 m<sup>2</sup>, while the mass is 3,410.0 kg.

Then, each osculating state was transformed into the mean state, which can be directly used as the initial conditions of HEOSAT. Fig. 1 shows the evolution of the osculating and mean eccentricities from November 2021 to November 2022.

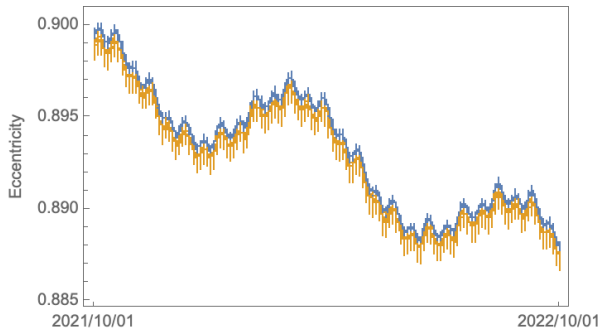


Fig. 1. Yearly evolution of the osculating and mean eccentricities in blue and brown colours, respectively.

These mean stages were propagated over 100 years. The results of this analysis are shown in Fig. 2 in terms of the year of re-entry of each initial condition. Red shows the re-entry between 1.5 and 6 years, green between 6 and 10 and magenta between 10 and 20 years. As can be observed, the red zone mainly appears between December 2021 and January 2022, whereas the green zone shows a periodic pattern.

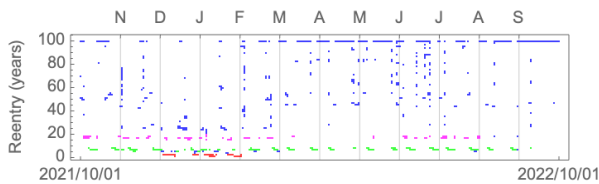


Fig. 2. Year of re-entry for each initial condition

Fig. 3 shows the histogram of the INTEGRAL re-entry times. The number of initial conditions that do not re-enter is 194,097, representing 36.9% of the cases. In 159,903 re-entry cases between 5 and 10 years, it is approximately 30.4%, while 66,738 cases, about 12.7%, are between 15 and 20. The remaining 104,862 cases are distributed in the rest of the intervals. It is worth noting that no re-entry conditions are found between 10 and 15 years of age.

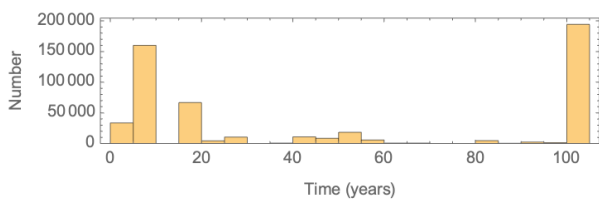


Fig. 3. Histogram of the INTEGRAL time to re-entry. The number of cases per interval is 33,758, 159,903, 0, 66,738, 4401, 10,723, 212, 765, 11,018, 8,590, 18,482, 5,964, 670, 761, 235, 90, 4,884, 461, 2,560, 1,288, 194,097, respectively.

### III. CONCLUSION

This paper presents preliminary numerical results. This

study evaluated the sensitivity of the INTEGRAL orbit by simulating its long-term evolution from November 2021 to November 2022. The propagation of the orbit was executed using HEOSAT. Notably, a significant portion, 63.1%, did re-enter within the studied timeframe, while only 36.9% did not re-enter.

### IV. ACKNOWLEDGMENTS

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